Generating Multivariate Normals

- How do you generate random draws from a multivariate normal?

$$N_d(\mu, \Sigma)$$

- Easy if Σ is diagonal: each component is independent, so draw each separately.
- Could modify Box–Muller method for a bivariate normal with $\rho \neq 1$.
- General methods involve matrix factorization.

Cholesky Decomposition

- The Cholesky decomposition is a way of factoring a non-negative definite matrix into a product of a lower triangular matrix and its transpose.
- $\Sigma = AA^T$
- It is a sort of matrix "square root"
- Routines exist in both IMSL and LAPACK

Generating Multivariate Normals

1. Factor $\Sigma = AA^T$
2. Draw d independent standard normals $(z_1, \ldots, z_d) = z$
3. Let $X = \mu + Az$
4. Then $X \sim N_d(\mu, \Sigma)$

`multnorm.c`, `multnorm2.c`

MCMC for Linear Regression

- For the standard model $y|\beta, \sigma^2, X \sim N(X\beta, \sigma^2 I)$
- with the standard noninformative prior $P(\beta, \sigma^2|X) \propto \frac{1}{\sigma^2}$
- the posterior is $\beta|\sigma^2, y \sim N(\hat{\beta}, V_\beta \sigma^2)$
- where $\hat{\beta} = (X^T X)^{-1} X^T y$ and $V_\beta = (X^T X)^{-1}$
- and $\sigma^2|y \sim Inv - \chi^2(n-k, s^2)$
- where $s^2 = \frac{1}{n-k}(y - X\hat{\beta})^T (y - X\hat{\beta})$

`linreg.mcmc.c`