

TATCGCATTTCAGCTA
CTTCGCATTTCGGCTAT
GGCTAACTTCGGCACA
GGCCCACTTTTACCG
TCTTATGTTTCCCCCG
ACTAGGATTTGGGAAC
CTCGGACTTTAACAAC
TATAAAGTTTGCGCGC
TATACCCTTTCACTTC

Two homologous sequences whisper ...
a full multiple alignment shouts out loud.

A. Lesk

The sum of pairs score:

Given a score system for pairwise alignment ...
... gaps are treated like any other character.

The sum of pairs score of one column is the
sum over all possible pairwise comparisons
in this column.

$$S(C) = \sum_{i < j} S(C(i), C(j))$$

Example: match =1, mismatch=0, space=-1

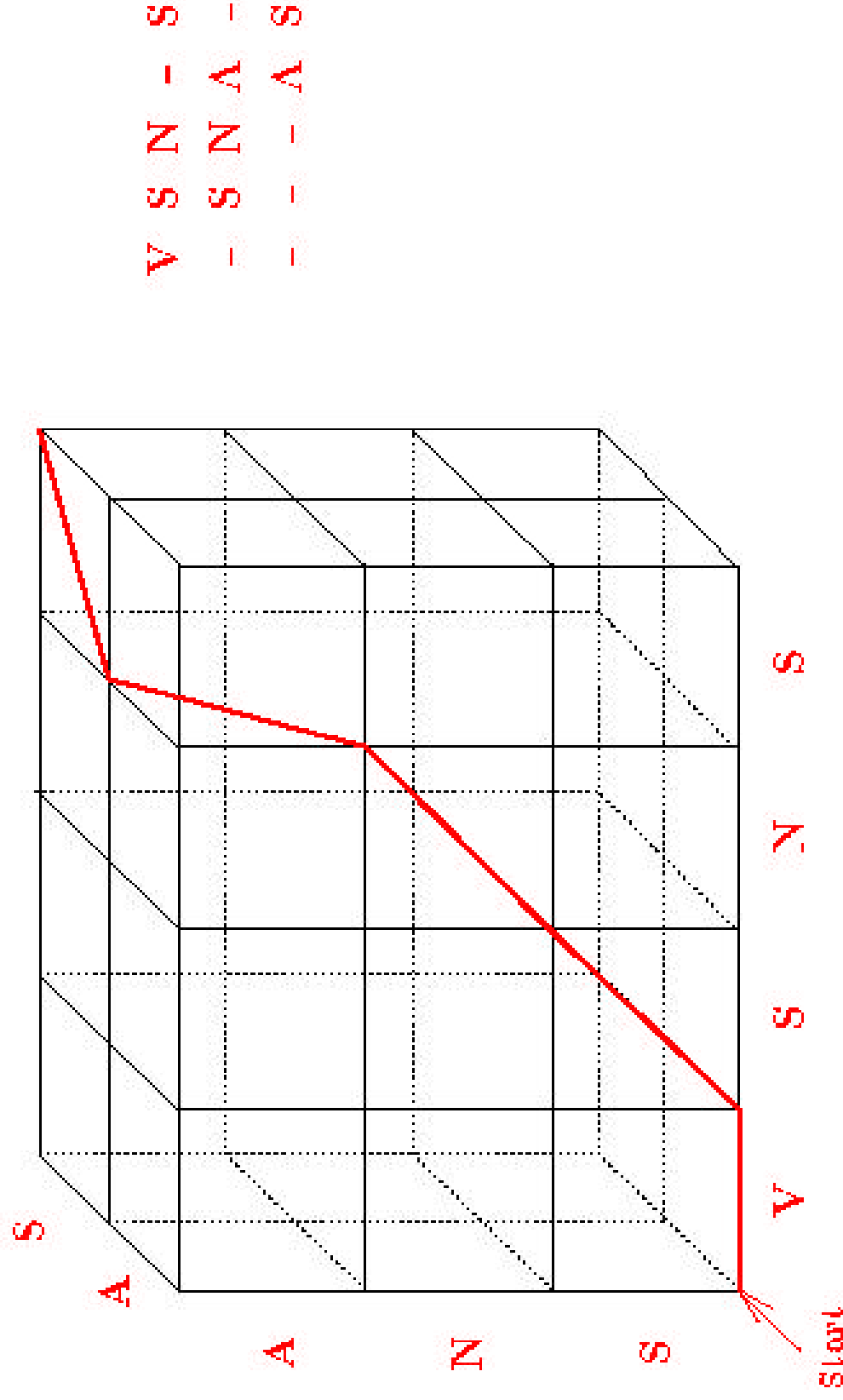
$$C = \begin{matrix} & T & A & - & T \\ \begin{matrix} T \\ A \\ - \\ T \end{matrix} & & & & \end{matrix}$$
Score=0-1+1-1+0-1=-2.

The score of the alignment is the sum of
the scores of the columns.

Three Protein Alignment

(Murata, Richardson & Sussman)

Figure 1: Alignment Path for 3 Sequences.



Time complexity exponential in the number of sequences:

$$O(g(n_1, \dots, n_k)^k)$$

NP-complete problem (Wang & Jiang 1994)

....forget it ...

... we had enough dynamic programming anyway.

Instead:

Construct multiple alignment from pairwise alignments.

Projected pairwise alignments

```
1  KL-ATYMKLSC
2  KL--TYKKL--
3  KL--TYKADSA
4  KLLYEYMKLS-
```

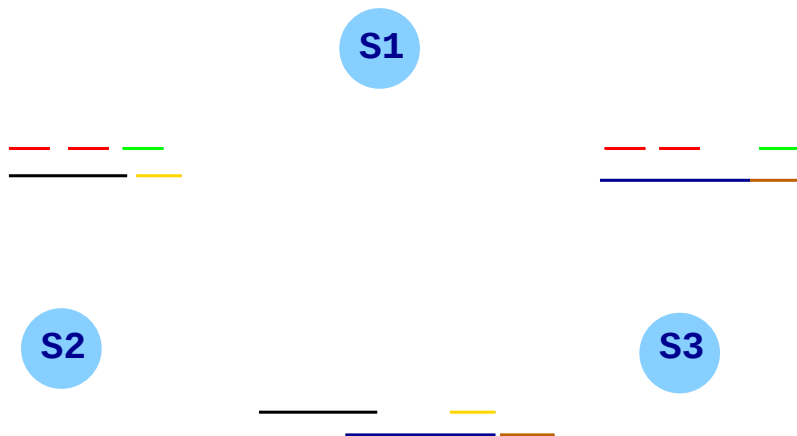
Projected alignment of 1 and 2

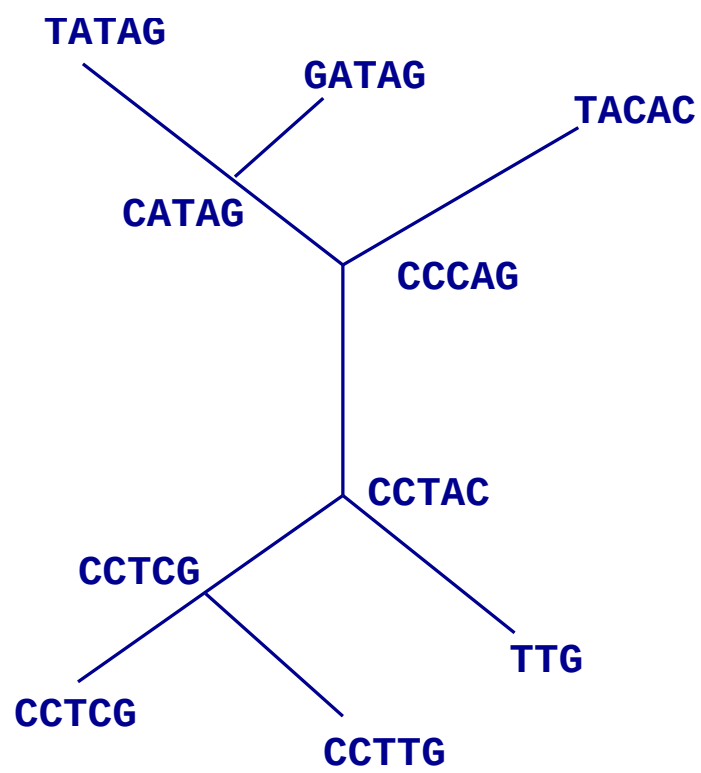
```
KLATYMKLSC
KL-TYMKLSC
```

copy the lines and delete opposite
spaces

The projections of an optimal multiple alignment (sum of pairs) need not be optimal with respect to the pairwise scores ...

... and it is in general not possible to merge optimal pairwise alignments to a consistent multiple alignment.





Tree alignment

The multiple alignment is constructed one string at a time.

1. Align two adjacent sequences say S1 and S2 (pairwise alignment)
2. Take the next sequence S3, it should be adjacent to either S1 and S2 ... assume it is connected to S1.
S1 might contain some gaps from the previous alignment, ...leave them in and align this gapped sequence to S3. Opposite gaps have a score of zero. (pairwise alignment)
Once a gap, always a gap
3. Repeat until all sequences are aligned.

If we project this multiple alignment to pairwise alignments, all pairwise alignments from sequences that are adjacent in the tree are optimal.

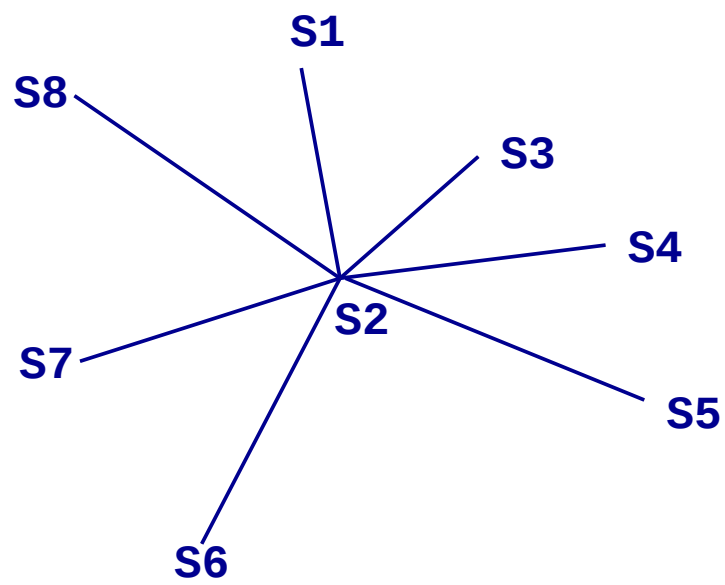
Great... but which tree do we choose ?

Score of a tree:
Sum of all pairwise alignment scores along edges of the tree.

Choose tree with optimal score among all possible trees ...

... the problem is NP-complete again.
Forget the optimal tree...

Star alignment



Choose center sequence S_i , such that S_i maximizes

$$\sum_{i \neq j} s(S_i, S_j)$$

When distances are used and the triangle inequality holds:

The sum of pairs score of the star alignment is less or equal two times the optimal score.

For a proof see Gusfield's book.

Scoring a multiple alignment

1. The positions are not all the same and should not be treated all the same
some are more conserved than others ...
these should determine the alignment.

2. The sequences are not independent, but related by a phylogenetic tree.

The sum of pairs score does not care at all ...

Another Problem with the sum of pairs score:

Alignment of N sequences.

There is a certain position with only L (leucine) for some important functional reason.

$s(L, L) = +5$

SP-Score: $5 N(N-1)/2$

By an alignment error we get 1 G (glycine) in this position and only N-1 L's.

$S(G, L) = -4$

SP-Score: 9 (N-1) less

Relative Loss: $9(N-1)/(5N(N-1)/2) = 18/5N$

The more L's we observe the more evidence we have that the G is wrong.

However, the relative loss in SP-Score is decreasing for a large number of sequences.

Pairwise comparison table

Calculate all pairwise alignment scores
and arrange them in a table

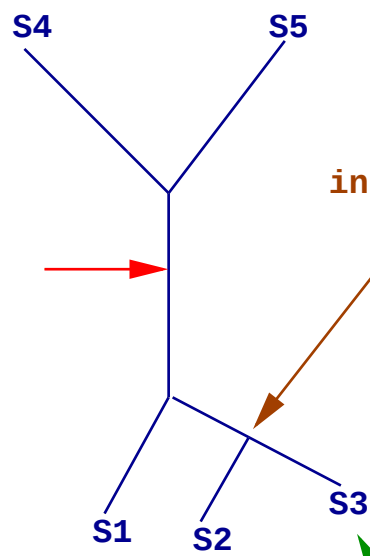
	S1	S2	S3	S4	S5
S1	●	-10	-5	4	-2
S2	-10	●	25	-8	0
S3	-5	25	●	-11	9
S4	4	-8	-11	●	-1
S5	-2	0	9	-1	●

Convert all score into distances ...

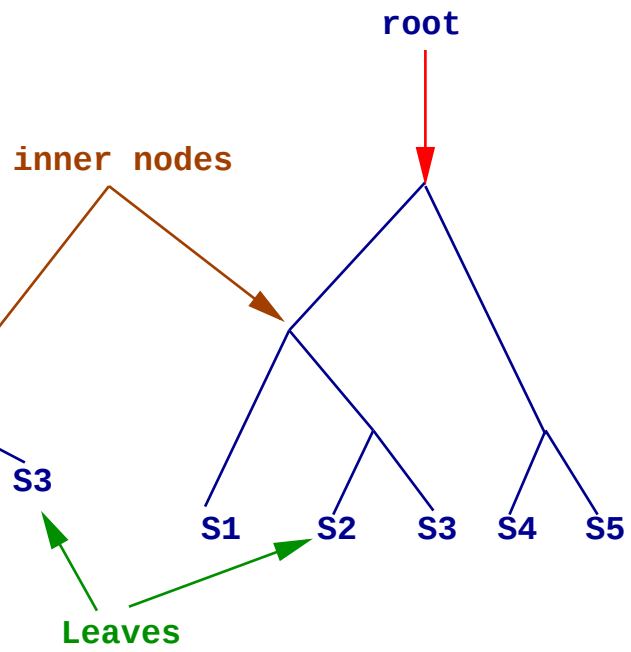
1. Feng-Doolittle: $D = -\log(S - S_{\text{rand}}) / (S_{\text{max}} - S_{\text{rand}})$
2. Model based distances

log det formular
maximum likelihood

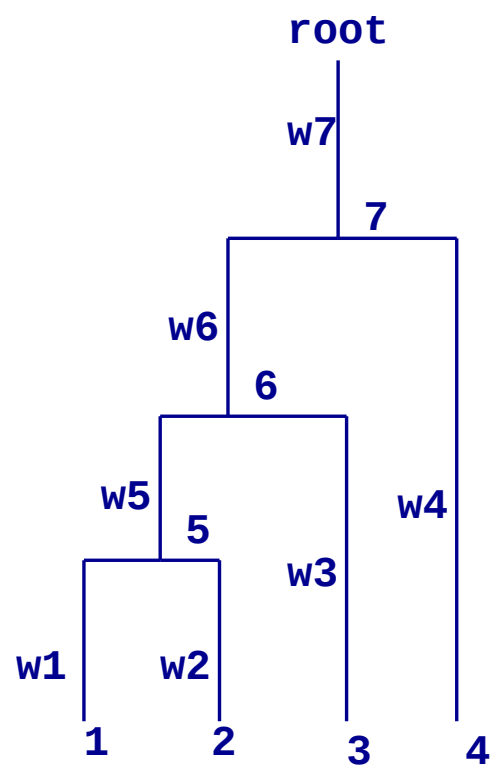
Unrooted Tree



Rooted Tree



Molecular clock:
Edge Lengths represent
"time of divergence"
The clock ticks at
a constant rate along all
paths in the tree.



Guide Trees by hierarchical clustering

1. Choose the pair with minimal distance and merge it to a new single unit. This unit consisting of two sequences is equivalent to all remaining sequences
2. Update the distance table by calculating the distance of this unit to all other units in the table.
3. repeat until there is only a single unit

This results in a rooted tree with sequences representing leaves.

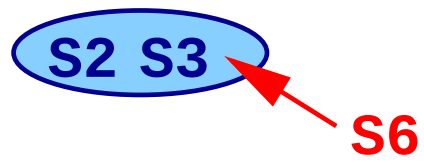
S1

S4

S5

S2

S3

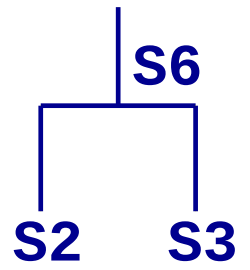


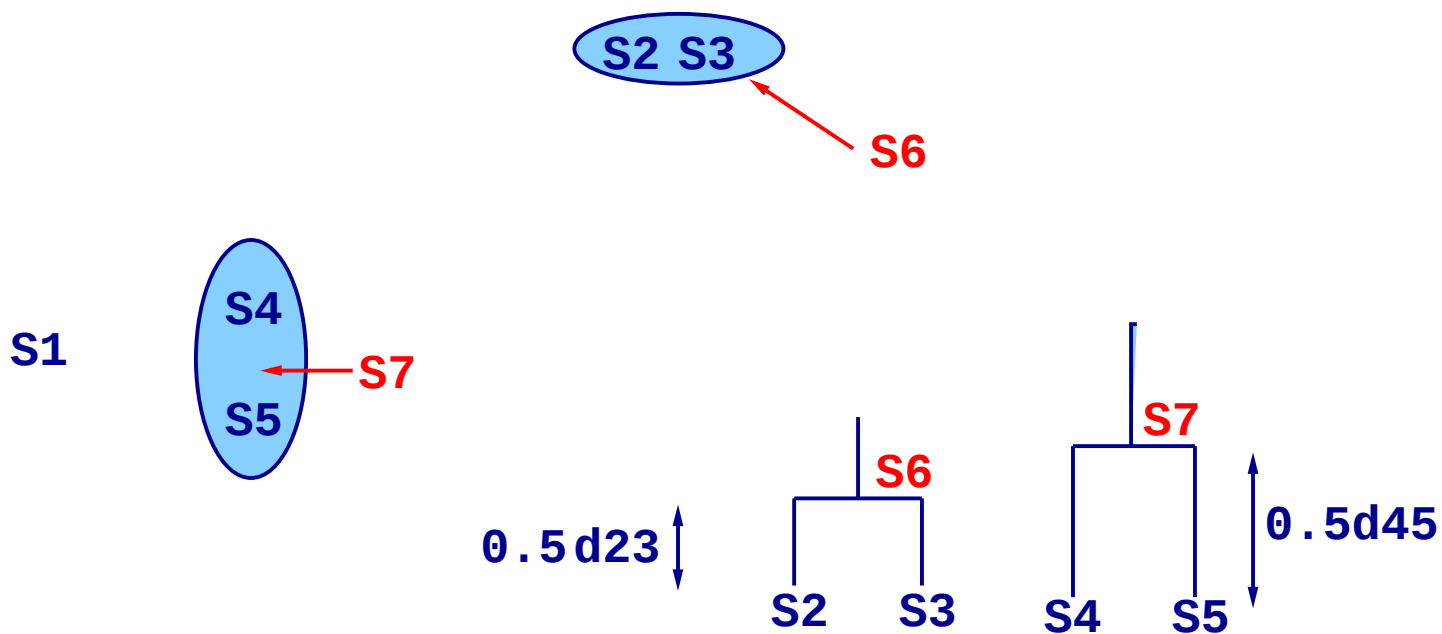
S1

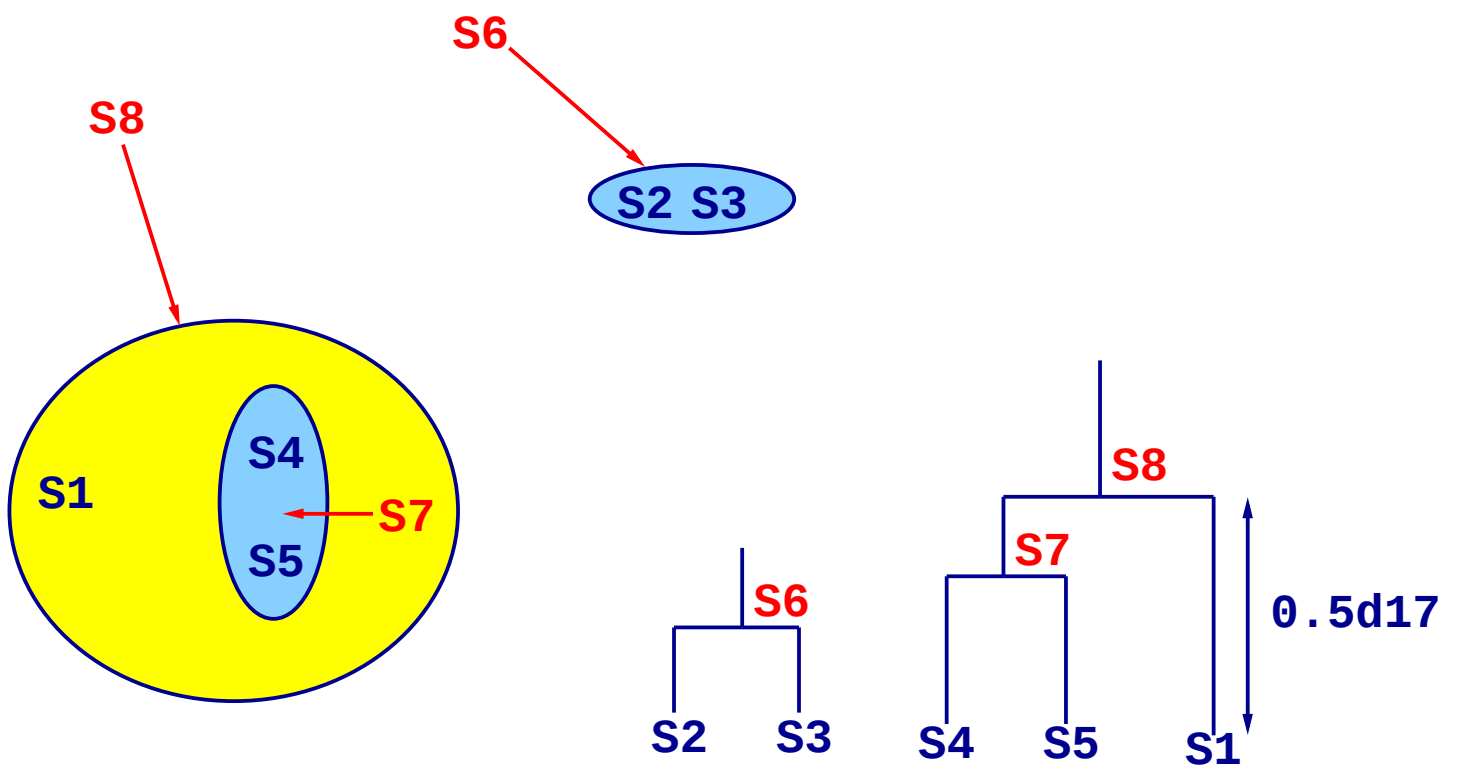
S4

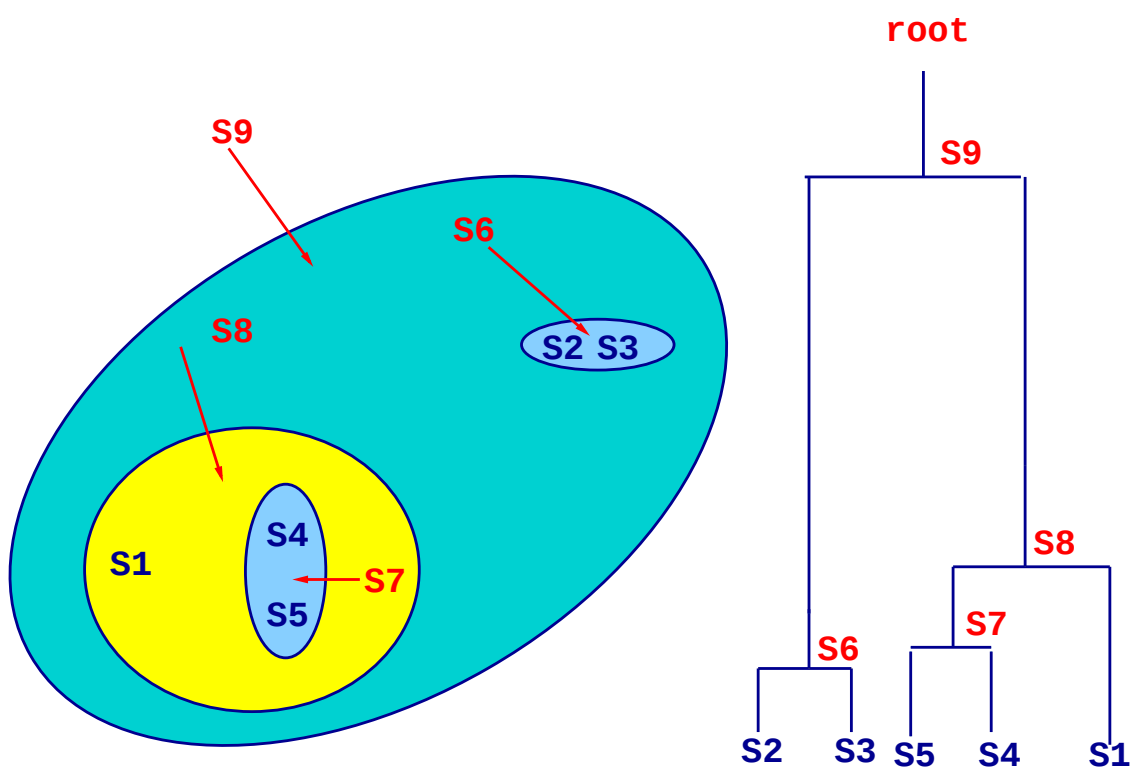
S5

0.5 d₂₃ 

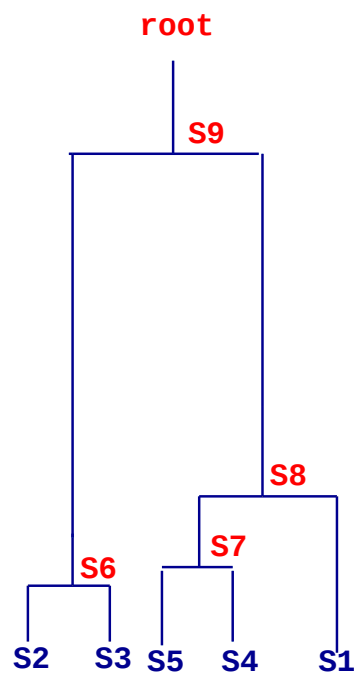








The Guide Tree



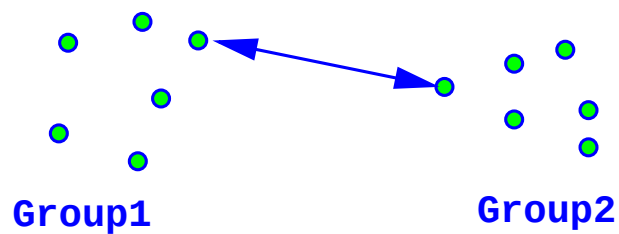
How do we calculate the score of groups of sequences?

Group1: $S_1, \dots, S_n$

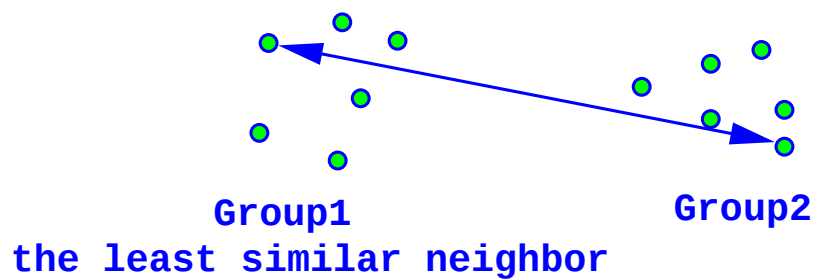
Group2: $T_1, \dots, T_m$

Single linkage: $S(\text{Group1}, \text{Group2}) = \max_{i,j} S(S_i, T_j)$

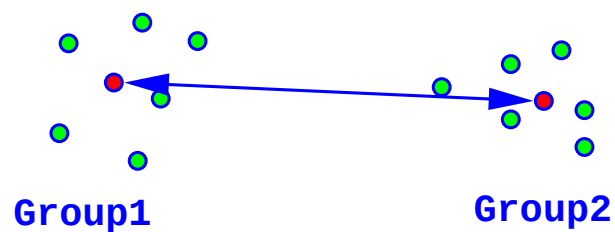
the most similar pair of sequences

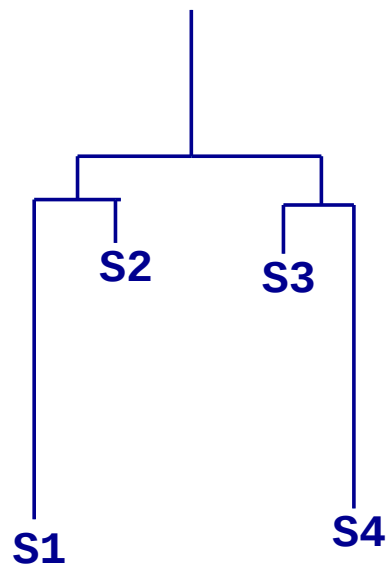


Total linkage: $S(\text{Group1}, \text{Group2}) = \min_{i,j} S(S_i, T_j)$

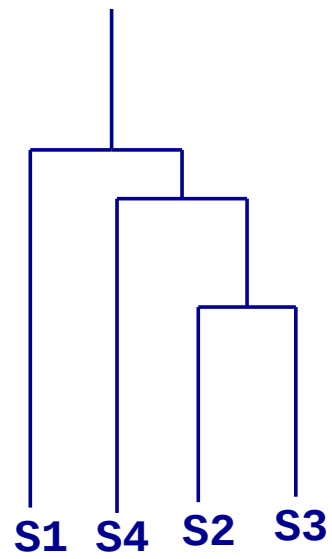


Average linkage: $S(\text{Group1}, \text{Group2}) = \text{mean } S(S_i, T_j)$





**Phylogenetic
Tree**



Guide Tree

Progressive Alignment

Build partial multiple alignments using only a subset of the sequences.

Start with the most similar sequences

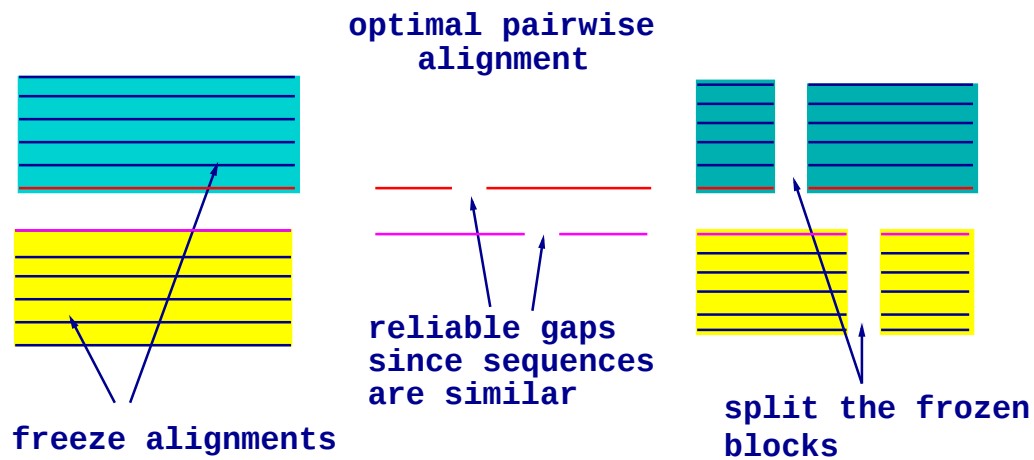
Join partial alignments using the "once a gap always a gap strategy"

The order in which sequences are included is determined by a Guide Tree

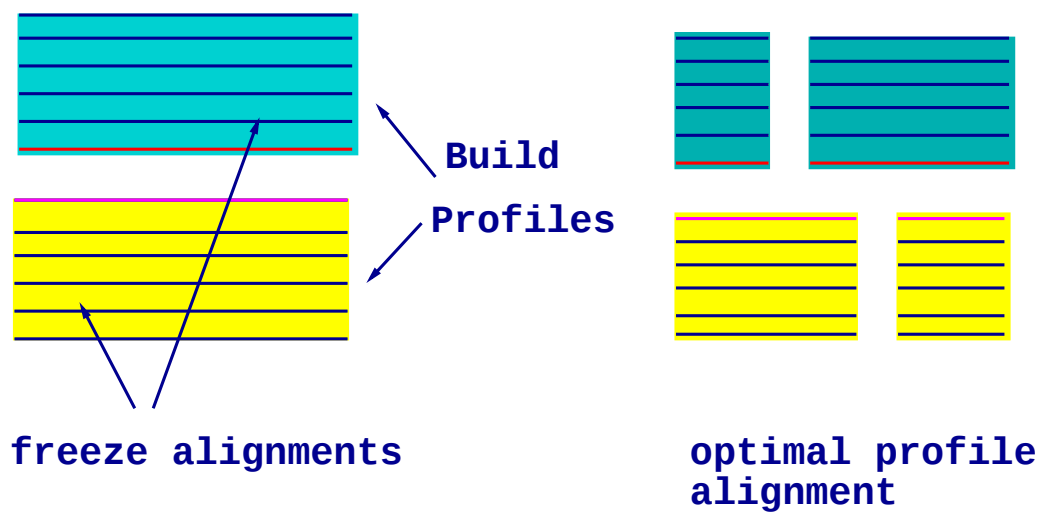
Feng Doolittle 1987

Join two partial multiple alignments
according to the optimal pairwise
alignment of the two most similar
sequences in the partial alignments.

once a gap always a gap



Progressive Profile Alignment



```
ATTCAGGGCATC
ATTCGGAGCATC
ATTCGGAGCATC
ATTCGGAGCATC
ATTCGGAGCATC
TATCCCAAGT
CACTGCACCCAT
```

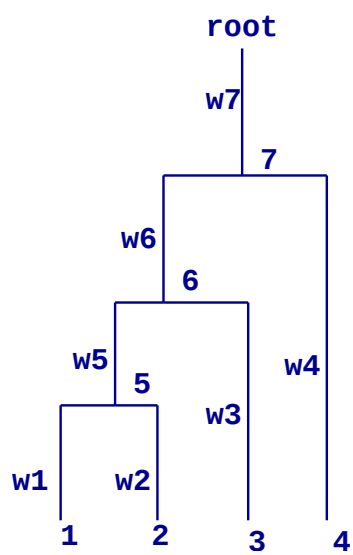
This set of very similar sequences will dominate the alignment.

Normally we do not have a random sample of family members. There are subfamilies that caught more attention in science than others ... hence, more data from this families is available.

Idea: Introduce sequence weights and limit the influence these clusters of similar sequences have in the alignment.

Rank them down ... and protect the rights of minorities.

Progressive Profile Alignment gives us a rooted tree with edge weights.



How do edge weights translate into weights for the sequences?

Gernstein, Sonnhammer and Chothia:

Work up the tree from the leaves to the root, incrementing the weights.

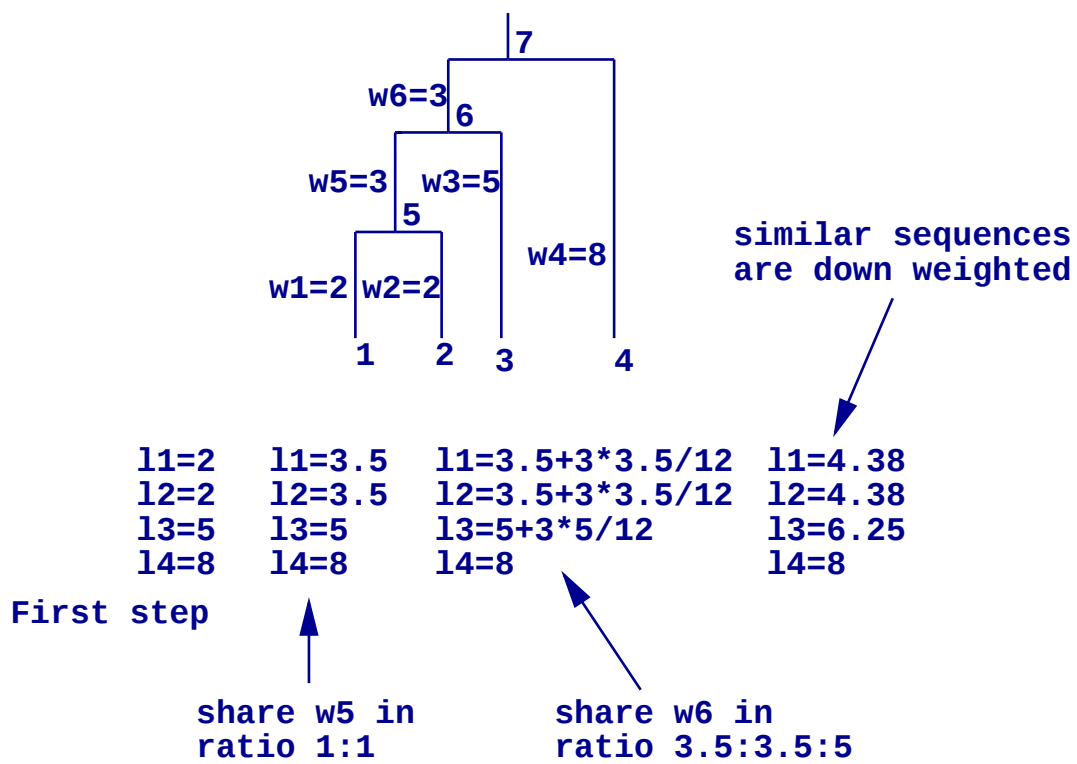
Let w_i denote edge weights and l_i denote sequence weights.

- (1) Assign to each sequence the weight of the edge right above it.

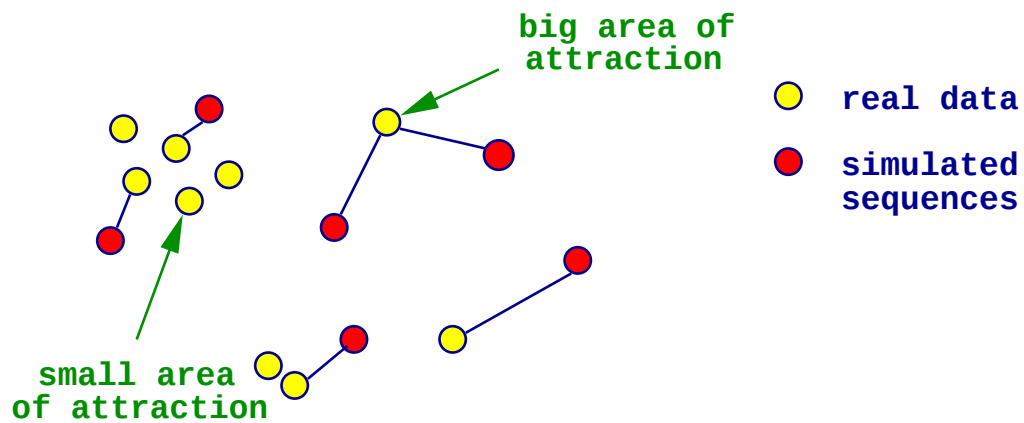
$$l_i = w_i$$

- (2) Now suppose node n has been reached
Share the weight w_n among all the sequences that are below n and increase their weights accordingly

$$\Delta l_i = w_i \cdot l_i / \sum_{k \text{ below } n} l_k$$



... or weights without a tree:



Simulate thousands of sequences from a profile type model and identify the sequence in the family that is most similar. Increase the count of this sequence by one. Use counts as weights.

We need sequence weights to calculate the **multiple alignment**

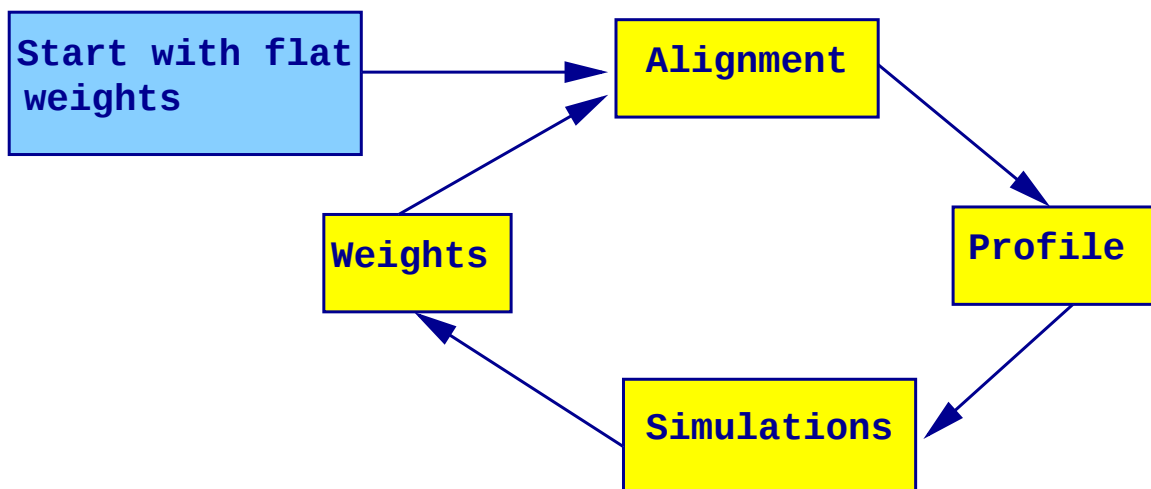
We need simulations to calculate weights

We need a model for the simulations

We need a profile for the model

We need a **multiple alignment**
for the profile

Iterative updating of the components.



Check convergence!

Other weighting methods

- Gaussian branching processes (used in ClustalW)
- Maximum Discrimination (used by hidden Markov models)
- Maximum Entropy

see Durbin et al.

...along the same lines:

Early decisions might be wrong since there was little profile information available at this time.

Remove the first sequence from the alignment and realign it to the almost full alignment using sequence to profile alignment.

Continue with the second, third ,... sequence.

Iterate for some time.

(not implemented in ClustalW)