## Some Hints for HW3

Several students told me about problems in resampling $x_t$. The acceptance rate in the rejections sampling seems very low.

Here are two alternative suggestions. Denote with $f(x_t) = p(x_t|x_{-t}, a, b, V, W, y)$ the conditional posterior distribution for $x_t$. We can evaluate $f(x_t)$ pointwise, i.e., for any given $x_t$. Let $F(x_t)$ denote the c.d.f. for $f(\cdot)$.

### Inverse c.d.f sampling:

Generate a draw from $f(x_t)$ by the inverse c.d.f. method:

$$x_t = F^{-1}(u) \text{ where } u \sim U(0,1)$$

1. Define a grid $g_1, \ldots, g_M$ for $x_t$. Use, for example, $g_1 = -10, g_M = 10$ and $M = 100$.
2. Evaluate $f(x_t)$ on the grid:
   (a) Evaluate $\log f(x_t)$ on the grid. Denote $lf = (\log f(g_1), \ldots, \log f(g_M))$.
   (b) Rescale: $lf := lf - \max(lf)$.
   (c) $f = \exp(lf)$
3. Compute the (empirical) c.d.f. $F_j = \sum_{k=1}^{j} f_j$: Use the R (or S-plus) command `F <- cumsum(f)`
4. Generate a draw from $f$ by generating a uniform r.v. $u$ and comptuing the inverse c.d.f. transform:

```
u <- runif(1)
i <- (1:M)[F>u][1]
th <- g[i]
```

### Metropolis Chain

Instead of a Gibbs sampler use a Metropolis chain step for updating $x_t$. I.e., instead of drawing from $f(x_t)$ go through the following steps:

1. Generate a candidate $\tilde{x} \sim N(x_t, C)$ with, for example, $C = 0.1$ (play around a bit with different choices).
2. Evaluate
$$a = \min\left\{1, \frac{f(\tilde{x})}{f(x_t)}\right\}$$
3. 
$$x_t = \begin{cases} \tilde{x} & \text{with probability } a \\ x_t & \text{otherwise} \end{cases}$$

Instead of the $N(x_t, C)$ "probing distribution" you could use alternative probing distributions.