Midterm Make-up Examination #1

Mth 135 = Sta 104

Due Friday, 2001 October 26, 10:30am

This is an open-book exam. It is similar to the (closed-book) in-class Midterm Exam, but it is a little longer and the questions have changed in minor ways. A normal distribution table and a blank worksheet are attached to the exam. If you don't understand something in one of the questions feel free to ask me (e-mail is best), but please do not talk to anyone else about the exam. You may use a calculator but not a computer.

You must **show** your **work** to get partial credit. Unsupported answers are not acceptable, even if they are correct. Please give all numerical answers as fractions **in lowest terms** (simplify!) or as decimals correct to **four places**. It is to your advantage to write your solutions as clearly as possible, since I cannot give credit for solutions I do not understand. Good luck.

Cheating on exams is a breach of trust with classmates and faculty, and will not be tolerated. After completing the exam please acknowledge the Duke Honor Code:

I have neither given nor received any unauthorized aid on this exam.

Signature: ____________________

Print Name: ____________________

1.	/20
2.	/20
3.	/20
4.	/20
5.	/20
6.	/20
Total:	/120

Problem 1: An experiment consists of tossing a fair coin four times. Find the probability mass function (pmf) for each of the following random variables:

a) (10) X, the number of heads *minus* the number of tails

$f(x) =$

b) (10) Y, the number of heads before the first tail (if any) appears.

$f(y) =$

Problem 2: Three identical fair coins are thrown simultaneously; we note whether or not they all show the same face.

a) (10) What is the probability that the number X of throws needed before they all show **different faces** is **more than** two? Give exact numerical answer.

$\mathsf{P}[X > 2] =$ ______________________

b) (10) What is the probability that the number Y of times they all agree in the next 100 tosses is at least 30? Half credit for a correct sum or integral expression; full credit for normal-distribution approximation to four correct decimal places.

(10) $\mathsf{P}[Y \geq 30] \approx$ ______________________

Problem 3: The width W of a square is an exponentially distributed random variable with probability density function

$$f_W(w) = \begin{cases} e^{-w} & w > 0 \\ 0 & w \le 0 \end{cases}$$

Let A be the area of the square. Find:

a) (10) $\mathsf{P}[A > 10] =$ ________________
(Give numerical answer to four decimals)

b) (10) The pdf $f_A(a) =$ ________________

Problem 4:

a) (10) A monkey types each of the 26 letters of the alphabet exactly once. What is the probability that the word "UNC" appears somewhere in the string?

$\mathsf{P}[\texttt{UNC}] =$ ____________________

b) (10) How many monkeys would it take typing such strings independently to have a probability of at least 0.90 of having `UNC` appear at least once?

$\mathsf{P}[\texttt{UNC}] \geq 0.90$ if $n \geq$ ____________________

Problem 5: A hat contains $n = 4$ coins, three of which are fair (so that $\mathsf{P}[H] = 1/2$) and one of which is biased with $\mathsf{P}[H] = 3/5$. A single coin is drawn at random from the hat. The questions below are about this one coin; it is not replaced, and no other coin is drawn. Be sure to **simplify** your answers below. F and B denote the events that the coin is Fair and Biased, respectively, and H_i the event that it falls Heads on the i^{th} toss.

a) (5) On the first toss, it lands Heads. What is the (conditional) probability that it is a fair coin?

$\mathsf{P}[F \mid H_1] =$ ______________

b) (5) It is tossed a second time. What is the (conditional) probability that it will land Heads this time, too, given Heads on the first toss? (note it could be fair or biased)

$\mathsf{P}[H_2 \mid H_1] =$ ______________

c) (5) If it does land Heads on both the first and second tosses, what is the probability that it is a biased coin?

$\mathsf{P}[B \mid H_1 \cap H_2] =$ ______________

d) (5) Denote by Z the number of Tails shown by this coin before the first Head. Give the pmf or pdf $f(z)$ for Z (first decide if Z has a discrete or continuous distribution). Do **not** condition on the outcomes of any tosses. Give your answer for **all** $z \in \mathbb{R}$.

$f(z) =$ __________

Problem 6: The events A and B are independent with probabilities $\mathsf{P}[A] = 0.3$ and $\mathsf{P}[B] = 0.6$, respectively. Find:

a) (5) $\mathsf{P}[A \cup B] =$ ____________

b) (5) $\mathsf{P}[A \cap B] =$ ____________

c) (5) $\mathsf{P}[A \mid B] =$ ____________

d) (5) $\mathsf{P}[A^c \cup B] =$ ____________

Extra worksheet, if needed:

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz:$$

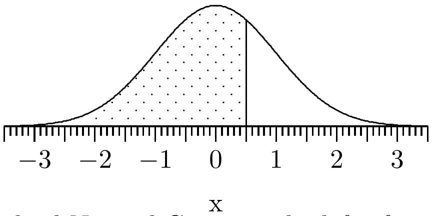

Table 5.1 Area $\Phi(x)$ under the Standard Normal Curve to the left of x.

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

$\Phi(0.6745) = 0.75$ $\Phi(1.6449) = 0.95$ $\Phi(2.3263) = 0.99$ $\Phi(3.0902) = 0.999$

$\Phi(1.2816) = 0.90$ $\Phi(1.9600) = 0.975$ $\Phi(2.5758) = 0.995$ $\Phi(3.2905) = 0.9995$