STA 104 Name:____________________________
MTH 135

Probability First Test

2:10-3:30 pm Thursday, 3 October 1996

This is a closed-book examination, so please do not refer to your notes, the text, or to any other books. If you don't understand something in one of the questions feel free to ask me, but please do not talk to each other. Please acknowledge the Duke Honor Code:

I have neither given nor received aid on this examination: ________________ .

You must **show** some **work** to get credit—*unsupported answers are not acceptable.* Attach any necessary work sheets to the exam before returning it; be sure to put your name on each page. It is to your advantage to write your solutions as clearly as possible, since I cannot give you credit for solutions I do not understand. Good luck.

1.	/20
2.	/20
3.	/20
4.	/20
5.	/20
	/100

STA 104
MTH 135

Probability First Test

1. Suppose that in a deck of 20 cards, each card has one of the numbers 1, 2, 3, 4, 5, with four cards of each number. Ten cards are chosen from the deck at random.

a. If the cards are drawn *with* replacement, what is the probability that each of the numbers 1, 2, 3, 4, 5 will appear exactly twice?

P = ______________

b. If the cards are drawn *without* replacement, what is the probability that each of the numbers 1, 2, 3, 4, 5 will appear exactly twice?

P = ______________

c. If the cards are drawn *without* replacement, what is the probability that the numbers X_1, X_2, X_3 on the first three cards drawn satisfy the strict inequality $X_1 < X_2 < X_3$?

P = ______________

2. Consider two events A and B such that $\mathsf{P}(A) = 1/3$ and $\mathsf{P}(B) = 1/2$. Find the probability of $A \cup B$ ("A or B") under each of the following conditions:

a. A and B are independent:

$\mathsf{P}(A \cup B) =$ ______________

b. A and B are disjoint:

$\mathsf{P}(A \cup B) =$ ______________

c. A implies B:

$\mathsf{P}(A \cup B) =$ ______________

d. $\mathsf{P}(AB) = 1/8$:

$\mathsf{P}(A \cup B) =$ ______________

e. $\mathsf{P}(A|B) = 1/8$:

$\mathsf{P}(A \cup B) =$ ______________

3. Two students A and B are both registered for MTH 135. If student A attends class 80% of the time and student B attends class 60% of the time, and if the absences of the two students are independent,

a. What is the probability that at least one of the students will be in class on a given day?

P = ____________

b. If *exactly* one of the students A and B is in class on a given day, what is the probability that A is present?

P = ____________

c. If *at least* one of the students A and B is in class on a given day, what is the probability that A is present?

P = ____________

STA 104
MTH 135

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4. Let X be a Poisson random variable with probability mass function

$$\mathsf{P}[X = k] = \frac{\lambda^k}{k!}e^{-\lambda}, \qquad k = 0, 1, 2, ...$$

Show your work in finding the following expectation:

a. $\mathsf{E}[2^X] =$ ______________

b. $\mathsf{E}[X!] =$ ______________

5. a. How many people are needed so that the probability that at least one of them has the same birthday as you is at least 1/2?

b. What assumptions did you make for 5a.?

c. Two balls are randomly chosen (without replacement) from an urn containing 8 white, 4 black, and 2 orange balls. Suppose that we win \$2 for each black ball selected and lose \$1 for each white ball selected. Let X denote our winnings. What are the possible values of X, and what are the probabilities associated with each value?