STA 104 Name:__________________________

MTH 135

Probability First Test

2:15-3:30 pm Thursday, 9 October 1997

You may use a one-sided 8.5 × 11" formula page but may not consult your books, notes, or neighbors. Please show your work for partial credit, and circle your answers. Points are awarded for *solutions*, not *answers*, so correct answers without justification will not receive full credit. Please give all numerical answers as decimals to four places. When you have finished, please sign the Duke Honor Code pledge.

I have neither given nor received aid on this examination.

____

1.	/20
2.	/20
3.	/20
4.	/20
5.	/20
	/100

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Probability First Test

1. In an unfair game Amanda draws a number at random from one to five (equally likely), while Blake draws a number at random from one to ten (also equally likely, and independent of Amanda's draw). The highest number wins; in the event of a tie, they both draw again. All draws are *with* replacement. Be sure to show your work below.

a. What is the probability that Amanda wins on the first draw?

P = ______________

b. What is the probability of a tie on the first draw?

P = ______________

c. What is the probability that Amanda wins the game (not necessarily on the first draw this time)? Why?

P = ______________

Probability First Test

2. *Pick a word at random from this sentence.*
You may assume that all the words are equally likely to be chosen.

a. What is the probability distribution of the length (number of letters) of the word chosen? (call the length X)

$\mathsf{P}[X = x] =$ ______________

b. What is the expected length of the word chosen?

$\mathsf{E}[X] =$ ______________

c. If the word is known to have more than four letters, what is the probability that it is the fifth word of the sentence, "random"?

$\mathsf{P}[\text{"random"} \mid X > 4] =$ ______________

3. Consider the following gambling game:

- I put \$1 on the table and toss a fair coin, over and over. Every time a Tail appears, I double the amount of money on the table; when a Head finally appears, the money on the table is yours. Thus if X denotes the number of Tails that appear before the first Head, then I pay you \$$Y = 2^X$.

a. What is the probability distribution of X, the number of tails before the first head?

$\mathsf{P}[X = k] =$ ____________ (for $k = 0, 1, 2, ...$)

b. What is the probability that I pay you more than \$100?

$\mathsf{P}[Y > 100] =$ ____________

c. A gambling game is called *fair* if the expectation of the net gain each play is zero— so that, in the long run, a player neither wins nor loses money. Let's try to make this game fair by charging you a fee of some amount \$$c$ to play. How large would c have to be for this game to be fair (i.e., for your expected net gain $\mathsf{E}[2^X - c]$ to be zero)?

$\mathsf{E}[Y] =$ ____________

d. Consider your answers to b. and c. above. Should a gambler be willing to pay \$100 to play the game? Why, or why not?

4.

a. Find the probability of the following events, for a random variable Z with the standard Normal distribution (mean $\mu = 0$, variance $\sigma^2 = 1$). Drawing little pictures can be *very* helpful for problems like these.

i) $\mathsf{P}[Z \in (-\infty, -0.5)] =$ ______________

ii) $\mathsf{P}[Z \in (-\infty, 0.5) \cup (1.5, \infty)] =$ ______________

b. Define a random variable X by $X = -2 + 4Z$ and find the following probability:

$\mathsf{P}[X \leq 4] =$ ______________

c. Define a random variable Y by $Y = \exp(Z)$ and find the probability:

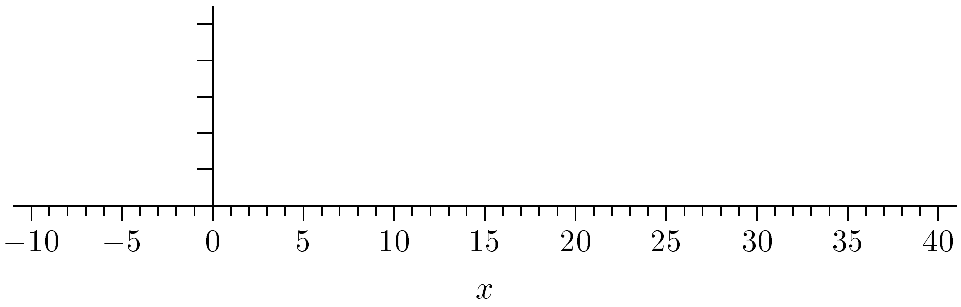
$\mathsf{P}[0.5 < Y \leq 2] =$ ______________

5. The probability density function of the lifetime of a certain type of transistors (measured in hours) is given by

$$f(x) = \begin{cases} c/x^2 & \text{for } x > 10 \\ 0 & \text{for } x \leq 10 \end{cases}.$$

a. Find the value of the constant: $c =$ ______________

b. Graph the probability density function using your value of c from above.



c. Find the probability that a transistor will last between 5 and 15 hours. Mark this probability in your plot in part b).

$\mathsf{P}[5 < X \leq 15] =$ ______________

d. Find the hazard function for the failure time X. Does it increase, decrease, both, or stay constant for $x > 10$?

$\lambda(x) =$ ______________

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Normal Distribution Table

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt:$$

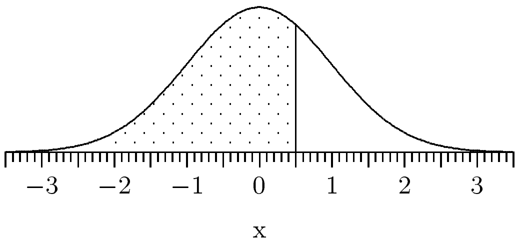

Table 5.1 Area $\Phi(x)$ under the Standard Normal Curve to the left of x.

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998