STA 104
MTH 135

Name:______________________________

Probability Second Test

2:15-3:30 pm Thursday, 20 November 1997

You may use a one-sided 8.5×11" formula page but may not consult your books, notes, or neighbors. Please show your work for partial credit, and circle your answers. Points are awarded for *solutions*, not *answers*, so correct answers without justification will not receive full credit. Please give all numerical answers as fractions in lowest terms or as decimals to four places. When you have finished, please sign the Duke Honor Code pledge.

I have neither given nor received aid on this examination.

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1.	/20
2.	/20
3.	/20
4.	/20
5.	/20
	/100

STA 104
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Probability Second Test

1. The random variable X has a uniform distribution on the interval $[0, 2]$, while Y has an exponential distribution with density function $f(y) = e^{-y}$, for $y > 0$; X and Y are independent. Be sure to show your work below, as you find:

a. $\mathsf{E}[X] =$ ____________ $\mathsf{P}[X > 2] =$ ____________

b. $\mathsf{E}[Y] =$ ____________ $\mathsf{P}[Y > 2] =$ ____________

c. $\mathsf{P}[Y > X] =$ ____________

d. The probability density function for $Z = \sqrt{Y}$:

$f_Z(z) =$ ____________

Probability Second Test

2. Let X and Y have joint density function

$$f(x, y) = \frac{1}{y} e^{-\frac{x}{y} - y}, \qquad 0 < x < \infty,\ 0 < y < \infty; \qquad f(x, y) = 0, \qquad \text{other } x, y.$$

a. Find $\mathsf{P}[Y > 2] =$ ______________

b. Find $\mathsf{P}[X > Y] =$ ______________

c. Find $\mathsf{P}[X > 2 | Y = 2] =$ ______________

3. Two coins fall "Heads up" with probabilities p_1 and p_2, independently, for some numbers $0 < p_1 < 1$, $0 < p_2 < 1$, and hence fall "Tails up" with probabilities $q_1 = 1 - p_1$ and $q_2 = 1 - p_2$, respectively. Both coins are tossed.

a. What is the probability that they show the same face?

b. If they *do* show the same face, what is the probability that the face they both show is Heads?

c. If we toss the two coins repeatedly until the first time they show *different* faces, what is the expected total number of Heads that will appear on the coins, up to (and including) the toss on which they differ?

4. Good news! An anonymous donor has agreed to give every Duke student a Thanksgiving Bonus. Bonus amounts (in dollars) will be independent random variables, each with density function $f(x) = .01e^{-.01x}$, for $x > 0$. Let X be the amount of *your* bonus, let S be the total amount of money the generous Donor must give, and assume for this problem that Duke has exactly 5,000 students enrolled. There is no need to do ANY integration for solving any part of this problem.

a. What is the probability distribution of X? Give its name and the value(s) of any parameter(s). Also give the mean, μ_X.

b. What is the probability distribution of S? Give its name and the value(s) of any parameter(s). Also give the mean, μ_S (which you can find even if you don't know the distribution).

c. Let Y be the number of students whose bonus is less than ten cents. Give the **exact** distribution of Y AND pick which (if any) distribution would be a good **approximation**: *Poisson*, *Geometric*, *normal*, or none of the above. If one of these is a good approximation, tell why and give the parameter(s); if not, explain why.

d. Maybe you are exceptionally lucky. Imagine that you talk to all your friends, one at a time, until you finally find someone with a bigger bonus than yours. What is the probability that you must ask k (or more) friends before finding one with a bigger bonus than yours? I.E., what is the probability that the number N of friends you must ask to find one with a bigger bonus than yours satisfies $N \geq k$? Note this question concerns the marginal distribution of N, *not* the conditional distribution given X.

$\mathsf{P}[N \geq k] =$ ______________

5. A new drug is being tested in a trial with 100 subjects, to discover the probability p that the drug is effective in reducing blood pressure. Of course there *may* be unexpected side-effects; let θ be the probability that a subject will experience an adverse drug reaction of some kind. The subjects are treated one at a time; if ANY ONE has an adverse reaction, the trial will be halted. Let S be the total number of subjects treated ($S = 100$ if there are no adverse reactions, but $S = s < 100$ if subject s has the first adverse reaction), and let X be the number of subjects who respond favorably to the blood-pressure treatment. Adverse reactions are independent of effectiveness—it is possible for the treatment to work for a subject who nevertheless experiences an adverse reaction.

a. What is the probability that the trial is *not* halted early?

$\mathsf{P}[S = 100] =$ ____________

b. What is the conditional probability of x subjects responding successfully, if there are no adverse reactions?

$\mathsf{P}[X = x | S = 100] =$ ____________

c. Are X and S independent? Why?

d. Give an expression for the *marginal* probability mass function for X; you need not evaluate the resulting sum (Hint: think about the distribution of S).