STA104: Solution to Homework 1

September 10, 2001

Problem (1)

(a) There are 26 possible letters and 10 possible digits. So the number of possible plates is $26^2 10^5 = 67,600,000$.

(b) If no letter or number can be repeated, the number of possible plates is $26 \times 25 \times 10 \times 9 \times 8 \times 7 \times 6 = 19,656,000$.

Problem (7)

(a) $6! = 720$

(b) $2! \times 3! \times 3! = 72$

(c) Within the group of the boys, there are $3! = 6$ possible ways. See the group of the boys as a whole, together with the 3 girls, there are $4!$ ways. So the total is $4! \times 3! = 144$.

(d) First put the boys in a line. OBGBGBO, "B" denotes Boys. In the "G" place there must be one and only one Girl. In The "O" place it's Optional if there's a girl or not. There are $3! = 6$ possible ways to locate the boys. And then there are $3 \times 2 \times 2 = 12$ ways to locate the girls. So the total is 72.

Problem (9)

$\binom{12}{6} \times \binom{6}{4} \times \binom{2}{1} \times \binom{1}{1} = 27,720$

Problem (13)

Nobody shakes anybody's hand twice. That means that the first person shakes hands with 19 people, the second person shakes hands with 18 "new" people (having already shaken hands with person number one), ... the last person shakes hands with no body "new". The total number of hand shakes are then $19 + 18 + 17 + \cdots + 1 + 0 = \frac{20*19}{2} = \binom{20}{2} = 190$.

Problem (16)

(a) $\binom{6}{2} + \binom{7}{2} + \binom{4}{2} = 15 + 21 + 6 = 42$.

(b) $\binom{6}{1}\binom{7}{1} + \binom{6}{1}\binom{4}{1} + \binom{4}{1}\binom{7}{1} = 6 * 7 + 6 * 4 + 4 * 7 = 42 + 24 + 28 = 94$

Problem (33)

(a) First we invest among the 4 possible opportunities with the minimal. Then we have 9 thousand dollars left. What we need to do now is to invest the 9 into 4 groups, and each group has no less than 0. This is equal to dividing 13 into 4 groups, and each group has 1 or more. This is actually to choose 3 bars in the 12 possible locations. So there are $\binom{12}{3} = 220$ possible strategies.

(b) If we don't invest in the first opportunity, then we have $\binom{13}{2}$ strategies, same for the second opportunity. Similarly for the third and fourth opportunities we have $\binom{14}{2}$ and $\binom{15}{2}$. To this we need to add the number of ways we can invest in all opportunities (i.e. from part (a)). So the total is $\binom{13}{2} + \binom{13}{2} + \binom{14}{2} + \binom{15}{2} + \binom{12}{3} = 2 * 78 + 91 + 105 + 220 = 572$

Exercise (2)

For the first experiment there are m possible outcomes. For each $i \in (1, 2, 3, \ldots, m)$ of these outcomes there are n_i outcomes for experiment two. So we simply need to add up all of the n_i's. The total number of outcomes $= \sum_{i=1}^{m} n_i$.

Exercise (13)

$$0 = (-1+1)^n = \sum_{i=0}^{n} \binom{n}{i} (-1)^i 1^{(n-i)} = \sum_{i=0}^{n} (-1)^i \binom{n}{i}$$

Extra problem

(a) Let the events A, B, C, D be the occurrence of a spill in 2002 at the respective sites (i.e. $A =$ Ackerby Farms, $B =$ Baker Hog Works, etc.). Then the sample space is $S = \{\emptyset$, ABCD, ABC, ABD, ACD, BCD, AB, AC, AD, BC, BD, CD, A, B, C, D$\}$. This has $2^4 = 16$ elements.

(b) An equally-likely assignment to each element in the sample space would imply that there is an equally-likely probability of there being a spill at each site, and that the probability of there being a spill at any site would be $\frac{1}{2}$. Having no knowledge of the industry and history of spills at specific sites and in the region, this would make for a simple model, assuming independence between sites and may be appropriate. However there are regional environmental factors that would suggest independence is not an appropriate assumption (such as floods, hurricanes, government funding restrictions, etc). In addition there are site specific factors that would be neglected if equally-likely probabilities we're assigned. Lastly, we'd all hope that the probability of there being no spills in 2002 be higher than $\frac{1}{16} = 0.0625$. Assigning the probability of $\frac{1}{2}$ to the event of a site having a spill maybe accurate, but we would hope it to be considerably less.