STA104: Solution to Homework 2

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Problem (1)

(a) $EF = E \cap F = \{(1,2),(1,4),(1,6),(2,1),(4,1),(6,1)\}$.

(b) $E \cup F =$ the set of events where the sum of the dice is odd, or there is at least one 1.

(c) $FG = F \cap G = \{(1,4),(4,1)\}$.

(d) $EF^C = E \cap F^C =$ the set of events where neither die is a 1, and the sum of the dice is odd.

(e) $EFG = FG = \{(1,4),(4,1)\}$.

Problem (6)

(a) $S = \{(1,g),(0,g),(1,f),(0,f),(1,s),(0,s)\}$.

(b) $A = \{(1,s),(0,s)\}$.

(c) $B = \{(0,g),(0,f),(0,s)\}$.

(d) $B^C \cup A = \{(1,s),(0,s),(1,g),(1,f)\}$.

Problem (9)

Let A represent the event of a customer having an American Express card, and V that they have a Visa card. Then $P(A \cup V) = P(A) + P(V) - P(A \cap V) = 0.24 + 0.61 - 0.11 = 0.74$, or 74% of the customers carry a credit card that is acceptable.

Problem (18)

There are $\binom{52}{2}$ ways to select two cards from a deck of 52. There are four ways to select an ace, and 16 ways to select either a ten, jack, queen, or king, and there are two ways to order the two cards that are selected. So the answer is $\frac{2*4*16}{\binom{52}{2}} = \frac{2*4*16}{52*51} = 0.0482$.

Problem (28)

Without replacement	With replacement
$(a)\quad \frac{\binom{5}{3}+\binom{6}{3}+\binom{8}{3}}{\binom{19}{3}} = 0.08875$	$(a)\quad \frac{5^3+6^3+8^3}{19^3} = 0.124$
$(b)\quad \frac{\binom{5}{1}\binom{6}{1}\binom{8}{1}}{\binom{19}{3}} = 0.2476$	$(b)\quad \frac{3!*5*6*8}{19^3} = \frac{6*5*6*8}{19^3} = 0.20994$

Problem (46)

We are interested in the event where at least two people in the room have the same birth-month. Using the law of complementary probabilities, look at the probability that none of the people in the room have the same birth-month, and subtract that from one. That is we want to find the smallest n such that $1 - \frac{12*11*10*\cdots*(13-n)}{12^n} \geq 0.5$. Simply by plugging values in for n we see that the probability of there being at least two of the same birth-months is 0.618 when there are $n = 5$ people in the room. Where as the probability is 0.427 when $n = 4$.

Problem (56)

Player B has the advantage in this game. If player A chooses spinner (a) then B can choose spinner (c). If A chooses (b) then B chooses (a). If A chooses (c) then B chooses (b). In each case B wins with probability 5/9.

Exercise (12)

$P(EF^C \cup E^C F) = P(EF^C) + P(E^C F) = P(E) - P(EF) + P(F) - P(EF) = P(E) + P(F) - 2P(EF).$

Exercise (20)

Assume that we have equally-likely probabilities for each of the events, call it p, where $0 \leq p \leq 1$. Then for all cases when $p \neq 0$, $\sum_{i=1}^{\infty} p = \infty$, and we do not have a distribution that sums to one, and this violates Axiom 2 (see page 29). Likewise, if $p = 0$, then $\sum_{i=1}^{\infty} p = 0$, and again Axiom 2 is violated. So this implies the there cannot be an infinite, albeit countable, number of events that are equally likely.

The question of all of these events having positive probability can be answered by looking at the Poisson distribution for example (there are many other examples aswell). This has p.m.f. of $p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$, for $x = 0, 1, 2, \ldots$. All of these are positive, and yet we know that the Poisson distribution is a real probability distribution.

Extra problem

We want to find the probability that none of the 39 students in this class has the same birthday as the instructor. If we consider that the probability that one student has a different birthday from the instructor is $\frac{364}{365}$, and that event is independent of any other student having a different birthday than the instructor, also $\frac{364}{365}$. Then the probability that none of the 39 students have the same birthday as the instructor is $\left(\frac{364}{365}\right)^n = \left(\frac{364}{365}\right)^{39} = 0.8985$