Homework 4 solutions

October 1, 2001

All problems are worth 4 points unless noted.

Problem 1 p.104

Let D be the event that the two outcomes are different.

$$P(6|D) = \frac{P(6,D)}{P(D)} = \frac{P(1^{st}=6, 2^{nd}\neq 6) + P(1^{st}\neq 6, 2^{nd}=6)}{\frac{5}{6}} = \frac{2\frac{1}{6}\frac{5}{6}}{\frac{5}{6}} = \frac{1}{3}$$

This could also have been solved by finding the reduced sample space - it is the same as asking for the probability that 6 is chosen when 2 of the numbers $1, 2, 3, 4, 5, 6$ are randomly chosen given that the outcomes differ.

Problem 20 p.106

a) $P(BBWW) =$

$$P(W|BBW)P(W|BB)P(B|B)P(B) = \frac{5}{12}\frac{7}{14}\frac{7}{16}\frac{9}{18} = \frac{35}{768} = 0.0455729$$

b) The probability of getting two black and two white marbles in the first four draws turns out to be the same for any of the six possible ways for it to happen, (i.e. $\{\{\text{BBWW}\}, \{\text{BWBW}\}, \{\text{WBWB}\}, \{\text{BWWB}\}, \{\text{WBBW}\}, \{\text{WWBB}\}\}$). Therefore we have

$$6 \times \frac{35}{768} = 0.273438$$

Problem 28 p.107

Let B and W be the events that the marble is black and white, respectively, and let B_i be the event that box i is chosen. Then,

$$P(B) = P(B|B_1)P(B_1) + P(B|B_2)P(B_2) = \frac{1}{2}\frac{1}{2} + \frac{2}{3}\frac{1}{2} = \frac{7}{12}$$

$$P(B_1|W) = \frac{P(W|B_1)P(B_1)}{P(W)} = \frac{\frac{1}{2}\frac{1}{2}}{\frac{5}{12}} = \frac{3}{5}$$

Problem 41 p.108

$$P(5^{th}|\text{heads}) = \frac{P(\text{heads}|5^{th})P(5^{th})}{\sum_i P(\text{heads}|i^{th})P(i^{th})} = \frac{\frac{5}{10}\frac{1}{10}}{\sum_{i=1}^{10}\frac{i}{10}\frac{1}{10}} = \frac{1}{11}$$

Problem 45 p.108

Let '+' be the event that someone tests positive, and let C be the event that someone has cancer. Then,

$$P(C|+) = \frac{P(+|C)P(C)}{P(+|C)P(C) + P(+|C^c)P(C^c)} = \frac{(.95)(.004)}{(.95)(.004) + (.05)(.996)} = 0.07089$$

Problem 50 p.108 (5 points)

These answers are not the only correct ones. Credit was given for having an answer, and backing it up with reasonable explanation.

a) Dependent. If eye color is not the determining factor in whether some one is hired, but is correlated to the determining factor, there would be dependence. For example a white, blue-eyed business woman may prefer to have a white secretary. This excludes all brown-eyed Asians, blacks, etc. who have brown eyes (generally) and significantly increases the chances that the secretary has blue eyes. In a similar manner, a black business woman may prefer a black secretary, both of whom would have the same (or at least similar) eye color.

b) Dependent. Wealth has an effect as a third variable. People that have enough money to buy a car generally also have enough money to have a phone. People who can't afford a phone certainly won't have enough to afford a car either.

c) Dependent, short people generally have less body mass than tall people, so we can assume some correlation between height and weight.

d) Dependent, since the United States is in the Western Hemisphere.

e) Dependence in this case can be seen in seasonal weather patterns. During rainy season, rain today has a strong effect on whether it rains tomorrow. Likewise during dry periods, lack of rain one day strongly effects whether it rains the next.

Problem 58 p.110

a)

$$P(\text{both hit}|\text{at least one hit}) = \frac{P(\text{both hit})}{P(\text{at least one hit})} = \frac{p_1 p_2}{1 - q_1 q_2}$$

b)

$$P(\text{Barb hit}|\text{at least one hit}) = \frac{p_1}{1 - q_1 q_2}$$

Problem 66 p. 112

a)

$$P(\text{carrier}|3\text{ without}) = \frac{\frac{1}{8}\frac{1}{2}}{\frac{1}{8}\frac{1}{2} + 1\frac{1}{2}} = \frac{1}{9}$$

b)

$$P(4^{th}|3\text{ without}) = \frac{1}{18}$$

Problem 76 p.114

Example $4j$ (Page 90) derives the 'gambler's ruin problem,' and this is just an application of it. Only we need to adjust x and N in order to use this result. Our \$10 is their zero, and our \$40 is their N, so we need to subtract \$10 from our x and N. The result follows:

$$\frac{1 - (9/11)^{15}}{1 - (9/11)^{30}} = 0.95303$$

Exercise 2 p.115

If $A \subset B$,

$$P(A|B) = \frac{P(A)}{P(B)}, \quad P(A|B^c) = 0, \quad P(B|A) = 1, \quad P(B|A^c) = \frac{P(BA^c)}{P(A^c)} = \frac{P(B) - P(A)}{1 - P(A)}$$

Exercise 10 p.116

The probability of flipping at least one heads in n flips is $1 - (1-p)^n$. We want n such that $1 - (1-p)^n \geq 1/2$. Solving for n gives $n \geq -\frac{\log(2)}{\log(1-p)}$.

Another Problem (5 points)

a) 2. Inherent to this problem is a fundamental question of Monty Hall's decision process, once you have chosen a door. We will assume that Monty has no preference as to which goat he reveals when he has the option of choosing between two goats (i.e. you initially choose the door with the car behind it, and Monty chooses between the two goats with equal probability). Consider the probability of winning the car, given that you always switch. That is $P(\text{car}|\text{switch}) = 2/3$. This is the same as the probability of initially choosing a door with a goat behind it since Monty Hall will not reveal the car when he opens a door. So if you switch, $2/3^{rd}$'s of the time you'll switch to the car. Likewise, the probability of choosing the car initially is $1/3$. So it doesn't matter which goat is revealed, if you switch then you've picked a goat $1/3^{rd}$ of the time. So you should always switch. (See the link 'MH' next to the link for this solution set for an interesting discussion and demonstration.)

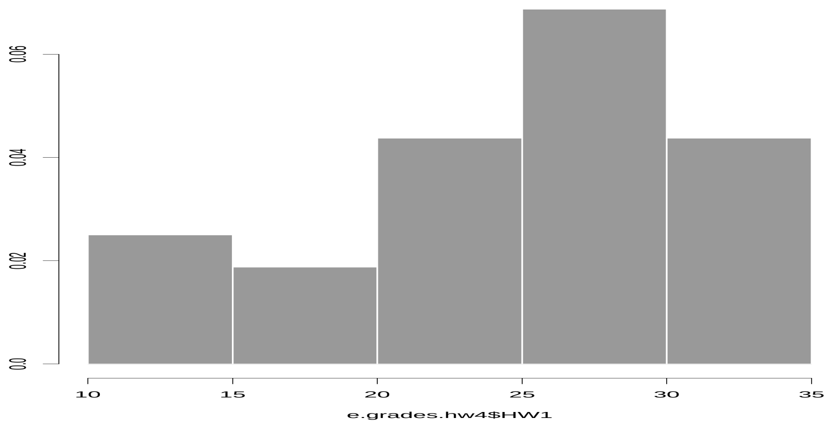
b) Again assuming that Monty picks a goat with equal probability if he has the choice between two goats, we can consider the following ordered pair (d_1, d_2). Where $d_1 = \{\text{door with car behind it}\} = \{\text{A,B, or C}\}$. And $d_2 = \{\text{door Monty opens, if he has a choice}\} = \{\text{A,B, or C} \mid d_2 \neq d_1\}$. With these events defined, we can write the sample space as the collection of the following six equally likely events

$$S = \{(A, B), (A, C), (B, A), (B, C), (C, A), (C, B)\}$$

Distributions for past home works

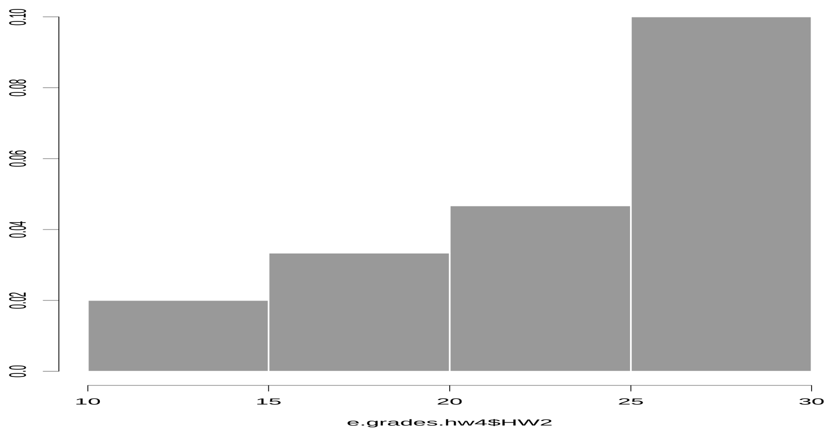
Several students have asked to see how the class is doing over all on the home works. Below is summary statistics and a histogram for each homework. In the future this will be included with each solution set.

Home Work 1



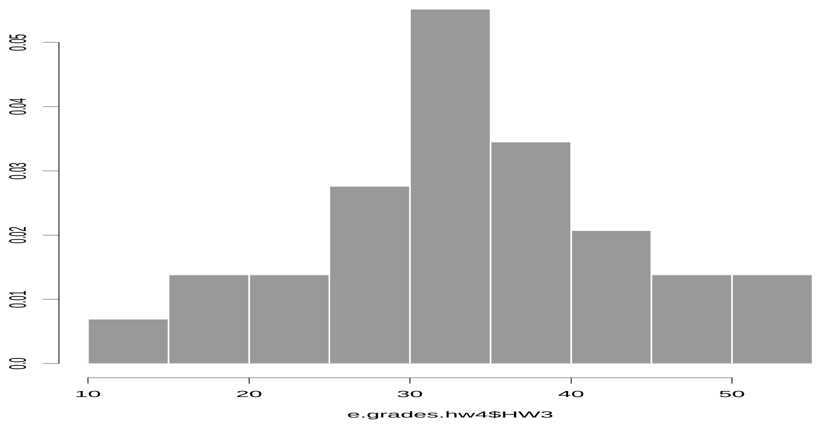
	Min	1^{st} Qu.	Median	Mean	3^{rd} Qu.	Max
Raw Score	12.00	21.75	26.50	25.03	30.00	32.00
Percentage	0.375	0.679	0.828	0.782	0.937	1.00

Home work 2



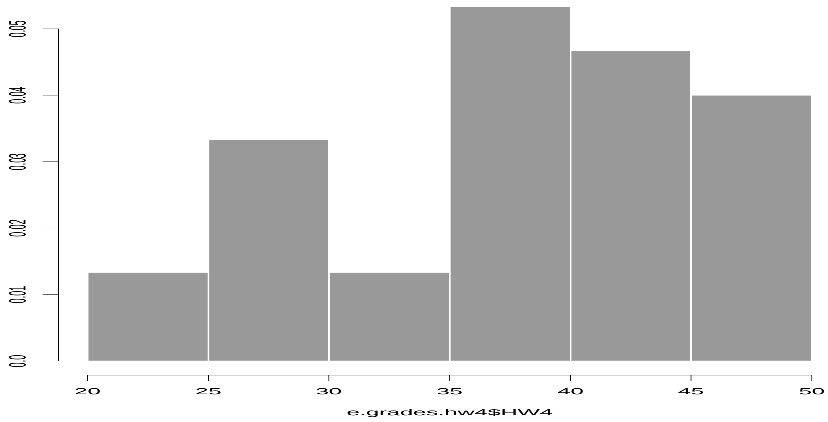
	Min	1^{st} Qu.	Median	Mean	3^{rd} Qu.	Max
Raw Score	11.00	20.25	25.50	23.63	28.00	30.00
Percentage	0.354	0.653	0.822	0.762	0.903	0.967

Home Work 3



	Min	**1^{st} Qu.**	**Median**	**Mean**	**3^{rd} Qu.**	**Max**
Raw Score	13.00	28.00	35.00	34.3440.00		52.00
Percentage	0.25	0.538	0.673	0.661	0.769	1.00

Home Work 4



	Min	**1^{st} Qu.**	**Median**	**Mean**	**3^{rd} Qu.**	**Max**
Raw Score	24.00	32.50	40.00	38.27	44.25	49.00
Percentage	0.48	0.65	0.80	0.765	0.885	0.980