Sta 104 Homework 5 solutions

October 8, 2001

All problems are worth 4 points unless noted.

Problem 1

a) c $\int_{-1}^{1}(1-x^2)dx = 1 \Rightarrow c = 3/4$

b) $F(x) = \frac{3}{4}\int_{-1}^{x}(1-x^2)dx = \frac{3}{4}\left(x - \frac{x^3}{3} + \frac{2}{3}\right), -1 < x < 1;$
$0, x < -1; 1, x > 1$

Problem 6 (5 points)

a) $E[X] = \int_0^\infty \frac{1}{4}x^2 e^{-x/2}dx = 2\int_0^\infty y^2 e^{-y}dy = 2\Gamma(3) = 4$ (or recognize that $X \sim Ga(2, 1/2)$)

b) By symmetry of $f(x)$ about $x = 0$, $E[X] = 0$ (or integrate)

c) $E[X] = \int_5^\infty \frac{5}{x}dx = \infty$

Problem 15 (5 points)

a) $1 - \Phi(\frac{5-10}{6}) = 1 - \Phi(-0.8333) = \Phi(0.8333) = 0.7977$

b) $\Phi(\frac{16-10}{6}) - \Phi(\frac{4-10}{6}) = 2\Phi(1) - 1 = 0.6827$

c) $\Phi(\frac{8-10}{6}) = \Phi(-0.3333) = 0.3695$

d) $\Phi(\frac{20-10}{6}) = \Phi(1.6667) = 0.9522$

c) $1 - \Phi(\frac{16-10}{10}) = 1 - \Phi(1) = 0.1587$

Problem 18

$0.2 = P\left(\frac{x-5}{\sigma} > \frac{9-5}{\sigma}\right) = P(Z > 4/\sigma)$ where Z is a standard normal. From the Splus `qnorm()` function, $P(Z < 0.8416212) = 0.8$ and so $0.8416212 = 4/\sigma$ or $\sigma = 4.752732$. That is, the variance is $(4.752732)^2 = 22.5884$.

Problem 26

a) $P(X \geq 525|\text{fair}) = P(Z > \frac{524.5-500}{\sqrt{(0.5)(0.5)(1000)}}) = 0.06062886$

b) $P(X < 525|\text{biased}) = P(Z < \frac{524.5-550}{\sqrt{(0.55)(0.45)(1000)}}) = 0.052521$

Problem 34

a) $P(X > 30|X > 10) = e^{-1.5}/e^{-0.5} = e^{-1} = 0.367879$

b) $P(X > 30|X > 10) = \frac{1/4}{3/4} = 1/3$

Problem 38

For both roots to be real the discriminant $(4Y)^2 - 4 \cdot 4(Y+2)$ must be ≥ 0. That is, we need that $Y^2 \geq Y + 2$. This implies that $Y \geq 2$ and so $P(Y \geq 2) = 3/5$.

Exercise 26

$\frac{X-a}{b-a}$

Exercise 28

$P(F(X) \leq x) = P(X \leq F^{-1}(x)) = F(F^{-1}(x)) = x, 0 < x < 1$

Another Problem (5 points)

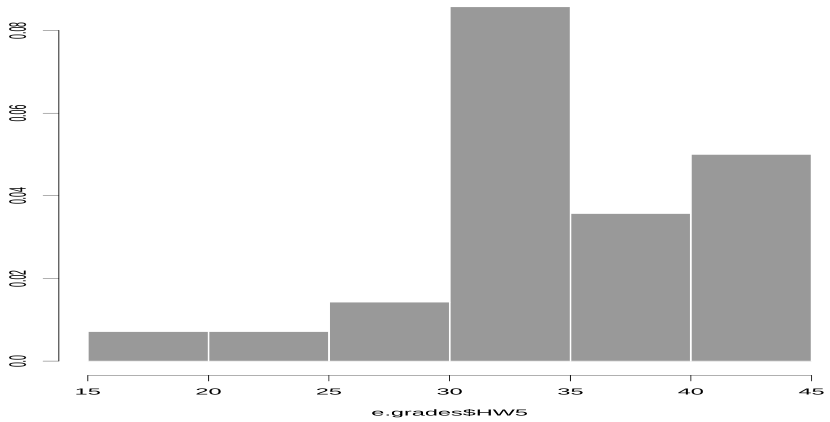
1) $P(X > 20) = \int_{20}^{\infty} \frac{10}{x^2} dx = \left[-\frac{10}{x}\right]_{20}^{\infty} = 0 - (-\frac{10}{20}) = \frac{1}{2}$

2) $P(X \leq x) = \int_{10}^{x} \frac{10}{t^2} dt = \frac{10}{10} - \frac{10}{x} = 1 - \frac{10}{x}, x \geq 10;\ 0, x < 10$

3) $\lambda(x) = \frac{f(x)}{1-F(x)} = \frac{10/x^2}{10/x} = \frac{1}{x}$, for $x > 10$, and 0 otherwise. This is a decreasing funcition over the interval $x > 10$

Distribution home work 5

43 total points possible



	Min	1^{st} Qu.	Median	Mean	3^{rd} Qu.	Max
Raw Score	17.00	31.75	34.00	34.54	40.25	42.00
Percentage	0.3954	0.7384	0.7907	0.8032	0.9361	0.9767