Sta 104 Homework 6 solutions

October 29, 2001

All problems are worth 4 points unless noted.

Problem 6 p.290/293*ff*

$N_2 \backslash N_1$	1	2	3	4
1	0.1	0	0	0
2	0.1	0.1	0	0
3	0.1	0.1	0.1	0
4	0.1	0.1	0.1	0.1

Problem 9 p.290/293*ff* (6 points)

(a) $\int_0^1 \int_0^2 \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) dydx = 1$

(b) $f_X(x) = \frac{6}{7}\int_0^2 \left(x^2 + \frac{xy}{2}\right) dy = \frac{6}{7}(2x^2 + x)$, for $0 < x < 1$, and 0 otherwise.

(c) $P(X > Y) = \int_0^1 \int_0^x \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) dydx = \frac{15}{56}$

(d) $P(Y > 1/2 | X < 1/2) = \frac{P(Y > 1/2, X < 1/2)}{P(X < 1/2)} = \frac{\int_{1/2}^2 \int_0^{1/2} \left(x^2 + \frac{xy}{2}\right) dxdy}{\int_0^{1/2} (2x^2 + x) dx} = \frac{69}{80}$

(e) $E[X] = \int_0^1 \frac{6}{7}(2x^3 + x^2)dx = \frac{5}{7}$

(f) $E[Y] = \int_0^2 \frac{6}{7}\left(\frac{y}{3} + \frac{y^2}{4}\right) dy = \frac{8}{7}$

Problem 20 p.290/293*ff*

(a) yes:
$f_X(x) = xe^{-x}$, for $0 < x < \infty$; $f_X(x) = 0$, otherwise.
$f_Y(y) = e^{-y}$, for $0 < y < \infty$; $f_Y(y) = 0$, otherwise.

(b) no:
$f_{X,Y}(\frac{2}{3}, \frac{1}{3}) = 0 \neq f_X(\frac{2}{3}) f_Y(\frac{1}{3})$

Problem 22 p.290/293ff

(a) No,
$f_X(x) = x + 1/2$, for $0 < x < 1$
$f_Y(y) = y + 1/2$, for $0 < y < 1$

$f(\frac{1}{4}, \frac{1}{4}) = \frac{1}{2} \neq \frac{9}{16} = f_X(\frac{1}{4}) f_Y(\frac{1}{4})$

(b) $f_X(x) = \int_0^1 (x+y)dy = x + 1/2$, for $0 < x < 1; f_X(x) = 0$, otherwise.

(c) $P(X+Y<1) = \int_0^1 \int_0^{1-x} (x+y)dydx = \int_0^1 [x(1-x) + (1-x)^2/2]dx = 1/3$

Problem 28 p.290/293ff

(a) $F_{\frac{X_1}{X_2}}(\alpha) = P(\frac{X_1}{X_2} < \alpha) = \int_0^\infty \int_0^{\alpha y} \lambda_1 e^{-\lambda_1 x} \lambda_2 e^{-\lambda_2 y} dxdy = \int_0^\infty \left(1 - e^{-\lambda_1 \alpha y}\right) \lambda_2 e^{-\lambda_2 y} dy = 1 - \frac{\lambda_2}{\lambda_2 + \alpha\lambda_1} = \frac{\alpha\lambda_1}{\lambda_2 + \alpha\lambda_1}$
$f_{\frac{X_1}{X_2}}(\alpha) = P(Z = \alpha) = \frac{\lambda_1\lambda_2}{(\lambda_2 + \lambda_1\alpha)^2}$

(b) $P(\frac{X_1}{X_2} < 1) = \frac{\lambda_1}{\lambda_1 + \lambda_2}$

Problem 41 p.290/293ff

(a) $p(x = 1|y = 1) = p(x = 2|y = 1) = 1/2$
$p(x = 1|y = 2) = 1/3; p(x = 2|y = 2) = 2/3$

(b) They are not independent since $p(x = 1)p(y = 1) = \frac{3}{8} \cdot \frac{1}{4} = \frac{3}{32} \neq \frac{1}{8} = p(x = 1, y = 1)$.

(c) $P(XY \leq 3) = p(1,1) + p(1,2) + p(2,1) = \frac{1}{2}$
$P(X + Y > 2) = p(1,2) + p(2,1) + p(2,2) = \frac{7}{8}$
$P(X/Y > 1) = p(2,1) = \frac{1}{8}$

Problem 42 p.290/293ff

(a) $f_{X|Y}(x|y) = \frac{xe^{-x(y+1)}}{\int xe^{-x(y+1)}dx} = (y+1)^2 xe^{-x(y+1)}, 0 < x;$ 0, otherwise (Gamma, $Ga(3, y+1)$).
$f_{Y|X}(y|x) = \frac{xe^{-x(y+1)}}{\int xe^{-x(y+1)}dy} = xe^{-xy}, 0 < y;$ 0, otherwise (Exponential, $Ex(x)$).

(b) $P(XY < a) = \int_0^\infty \int_0^{a/x} xe^{-x(y+1)}dydx = \int_0^\infty (1 - e^{-a})e^{-x}dx = 1 - e^{-a}$

$f_{XY}(a) = e^{-a}, 0 < a;$ 0, otherwise.

Exercise 5 p.296/300ff (5 points)

(a) For $a > 0$
$F_Z(a) = P(X \leq aY) = \int_0^\infty \int_0^{ay} f_X(x) f_Y(y) dx dy = \int_0^\infty F_X(ay) f_Y(y) dy$
$f_Z(a) = \int_0^\infty f_X(ay) y f_Y(y) dy$

(b) $F_Z(a) = P(XY < a) = \int_0^\infty \int_0^{a/y} f_X(x) f_Y(y) dx dy = \int_0^\infty F_X(a/y) f_Y(y) dy$
$f_Z(a) = \int_0^\infty f_X(a/y) \frac{1}{y} f_Y(y) dy$

If X is exponential with rate λ and Y is exponential with rate μ then (a) and (b) reduce to

(a) $F_Z(a) = \int_0^\infty \lambda e^{-\lambda a y} y \mu e^{-\mu y} dy = \frac{\lambda\mu}{(\lambda a + \mu)^2}$

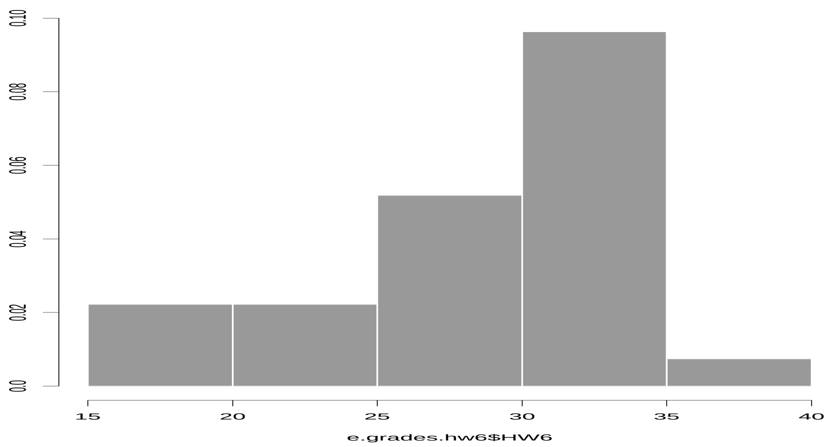
(b) $F_Z(a) = \int_0^\infty \lambda e^{-\lambda a/y} \frac{1}{y} \mu e^{-\mu y} dy$

Another Problem (5 points)

We Assume that where one coin lands doesn't have any bearing on where the others land, that is we assume that where these three coins land are independent and identically distributed. The center of each coin is a total distance from the center of the can between $[0, 2]$. So $P(R \leq r) = \frac{\pi r^2}{\pi 2^2} = \frac{r^2}{4}$, for $0 < r < 2$. Since these are independent, $F_{R_{(3)}}(r) = P(R_1 \leq r, R_2 \leq r, R_3 \leq r) = P(R_1 \leq r)P(R_2 \leq r)P(R_3 \leq r) = \frac{r^6}{64}, 0 < r < 2$. Then $f_{R_{(3)}}(r) = \frac{3r^5}{32}, 0 < r < 2$.

Distribution for Homework 6

40 **total points possible**



	Min	1^{st} Qu.	Median	Mean	3^{rd} Qu.	Max
Raw Score	15.00	26.5	31.0	29.4	34.5	36.0
Percentage	0.375	0.663	0.775	0.734	0.863	0.90