Sta 104 Homework 8 Solutions

November 12, 2001

All problems are worth 4 points unless noted.

Problem 1 p.379/372*ff*

Let X be the winnings. Then
$E[X] = 1/2\sum_{x=1}^{6}(2x + x/2)P(X = x) = \frac{105}{24} = 4.375$

Problem 21 p.379/372*ff*

(a) Let

$$X_i = \begin{cases} 1, & \text{if day } i \text{ is exactly three people's birthday} \\ 0, & \text{otherwise} \end{cases}$$

Then

$$E[X_i] = \binom{100}{3}\left(\frac{1}{365}\right)^3\left(\frac{364}{365}\right)^{97}$$

And

$$E[X] = E[\sum_{i=1}^{365} X_i] = \sum_{i=1}^{365} E[X_i] = 365\binom{100}{3}\left(\frac{1}{365}\right)^3\left(\frac{364}{365}\right)^{97} = 0.9301$$

(b) Let

$$X_i = \begin{cases} 1, & \text{if day } i \text{ is at least one person's birthday} \\ 0, & \text{otherwise} \end{cases}$$

Then

$$E[X] = E[\sum_{i=1}^{365} X_i] = \sum_{i=1}^{365} E[X_i] = 365\left[1 - \left(\frac{364}{365}^{100}\right)\right] = 87.575$$

Problem 31/27 p.379/372ff

$Var\left(\sum_{i=1}^{10} X_i\right) = 10 \cdot Var(X_1)$. Here

$$\begin{aligned} Var(X_1) &= E[X_1^2] - E[X_1]^2 \\ &= [1+4+9+16+25+36]/6 - (7/2)^2 \\ &= 35/12 \end{aligned}$$

and so

$$Var\left(\sum_{i=1}^{10} X_i\right) = 350/12 = 29.1667$$

Problem 54/50 p.379/372ff

Let R_i denote the return from the policy that stops the first time a value at least as large as i appears. Also, let X be the first sum, and let $p_i = P(X = i)$. Conditioning on X yields

$E[R_5] = \sum_{i=2}^{12} E[R_5|X = i]p_i$

$= E[R_5](p_2 + p_3 + p_4) + \sum_{i=5}^{12} ip_i - 7p_7$

$= \frac{6}{36}E[R_5] + 5(\frac{4}{36}) + 6(\frac{5}{36}) + 8(\frac{5}{36}) + 9(\frac{4}{36}) + 10(\frac{3}{36}) + 11(\frac{2}{36}) + 12(\frac{1}{36})$

$= \frac{6}{36}E[R_5] + \frac{190}{36}$

Hence, $E[R_5] = \frac{19}{3} = 6.\bar{3}$. In the same fashion, we obtain that

$$E[R_6] = \frac{10}{36}E[R_6] + \frac{1}{36}[30 + 40 + 36 + 30 + 22 + 12]$$

implying that

$$E[R_6] = \frac{170}{26} = 6.539$$

Also,

$$E[R_8] = \frac{15}{36}E[R_8] + \frac{1}{36}(140)$$

or,

$$E[R_8] = \frac{140}{21} = 6.\bar{6}$$

In addition,

$$E[R_9] = \frac{100}{16} = 6.25$$

And

$$E[R_{10}] = \frac{64}{12} = 5.\bar{3}$$

The maximum expected return is $E[R_8]$.

Problem 66/61 p.379/372*ff*

$E[X^2] = \frac{1}{3}(E[X^2|Y=1] + E[X^2|Y=2] + E[X^2|Y=3])$

$= \frac{1}{3}(9 + E[(5+X)^2] + E[(7+X)^2])$

$= \frac{1}{3}(83 + 24E[X] + 2E[X^2])$

$= \frac{1}{3}(443 + 2E[X^2])$ since $E[X] = 15$

Hence,

$$Var(X) = 443 - (15)^2 = 218$$

Problem 68/64 p.379/372*ff*

(a) $0.6e^{-2} + 0.4e^{-3} = 0.101116$

(b) $0.6e^{-2}\left(\frac{2^3}{3!}\right) + 0.4e^{-3}\left(\frac{3^3}{3!}\right) = 0.197884$

(c) $P(3|0) = \frac{P(3,0)}{P(0)} = \frac{0.6e^{-2}e^{-2}\left(\frac{2^3}{3!}\right)+0.4e^{-3}e^{-3}\left(\frac{3^3}{3!}\right)}{0.6e^{-2}+0.4e^{-3}} = 0.189033$

Another Problem (6 points)

(a) Using the geometric distribution as parameterized in the text, each of X_i, for $i = 2,3,4$ is a geometric distribution with means respectively $p = \frac{3}{4}, \frac{1}{2}, \frac{1}{4}$. So $E[X_i] = \frac{4}{3}, \frac{2}{1}, \frac{4}{1}$, again respectively. And of course $E[X_1] = 1$. So if X is the random variable we are interested in, that is the total amount of money Cary spends, $E[X] = \sum_{i=1}^{4} E[X_i] = 8.\bar{3}$

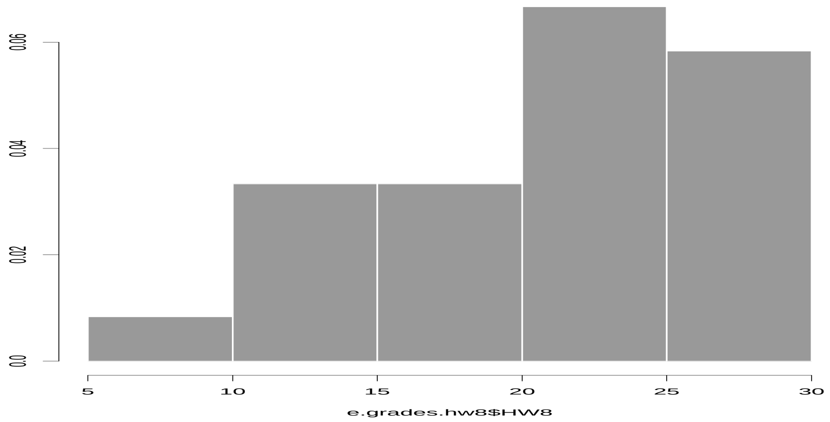
(b) $P(X \leq 5) = P(X=4) + P(X=5)$

Where $P(X=4) = P(X_2 = 1, X_3 = 1, X_4 = 1)$, and $P(X=5) = P(X_2 = 2, X_3 = 1, X_4 = 1) + P(X_2 = 1, X_3 = 2, X_4 = 1) + P(X_2 = 1, X_3 = 1, X_4 = 2)$

Using the correct geometric random variables, and assuming independence gives $P(X \leq 5) = 0.234375$.

Distribution for Homework 8

30 total points possible



	Min	1^{st} **Qu.**	**Median**	**Mean**	3^{rd} **Qu.**	**Max**
Raw Score	7.00	17.50	23.50	21.75	28.00	30.00
Percentage	0.2188	0.5469	0.734	0.6797	0.875	0.9375