Random Effects Model

or

Bayesian Hierarchical Analysis of Variance

Data:

$$x_{ij} \sim N(\mu_i, \frac{1}{\tau}) \text{ for } i = 1, \ldots, I \text{ and } j = 1, \ldots, n_i \ .$$

$$l(\mu, \sigma | X) \propto -\frac{\tau}{2} \sum_{i=1}^{I} \sum_{j=1}^{n_i} (x_{ij} - \mu_i)^2$$

Priors:

$$\pi(\mu_i | \psi, \omega) \sim N(\psi, \frac{1}{\omega})$$

$$\pi(\psi) \sim N(1, 1)$$

$$\pi(\tau) \sim \Gamma(1, 1) \propto \exp(-\tau)$$

$$\pi(\omega) \sim \Gamma(1, 1)$$

$$
\begin{aligned}
P(\mu,\psi,\tau,\omega|X) &\propto \tau^{n/2}\exp\left(-\frac{\tau}{2}\sum_i\sum_j(x_{ij}-\mu_i)^2\right) \\
&\times \omega^{I/2}\exp\left(-\frac{\omega}{2}\sum_i(\mu_i-\psi)^2\right)e^{-(\psi-1)^2/2}e^{-\tau}e^{-\omega}
\end{aligned}
$$

$$
\begin{aligned}
\mu_i|- &\propto \exp\left(-\frac{\tau}{2}\sum_j(x_{ij}-\mu_i)^2-\frac{\omega}{2}(\mu_i-\psi)^2\right) \\
&\sim N\left(\frac{\tau\sum_{j=1}^{n_i}x_{ij}+\omega\psi}{\tau n_i+\omega}\,,\ \frac{1}{\tau n_i+\omega}\right) \\
\psi|- &\propto \exp\left(-\frac{\omega}{2}\sum_i(\mu_i-\psi)^2-\frac{1}{2}(\psi-1)^2\right) \\
&\sim N\left(\frac{\omega\sum_i\mu_i+1}{\omega I+1}\,,\ \frac{1}{\omega I+1}\right)
\end{aligned}
$$

$$
\begin{aligned}
\tau|- &\propto \tau^{n/2} \exp\left(-\tau - \frac{\tau}{2}\sum_i \sum_j (x_{ij} - \mu_i)^2\right) \\
&\sim \Gamma\left(1 + \frac{n}{2}\,,\; 1 + \frac{1}{2}\sum_i \sum_j (x_{ij} - \mu_i)^2\right)
\end{aligned}
$$

$$
\begin{aligned}
\omega|- &\propto \omega^{I/2} \exp\left(-\omega - \frac{\omega}{2}\sum_i (\mu_i - \psi)^2\right) \\
&\sim \Gamma\left(1 + \frac{I}{2}\,,\; 1 + \frac{1}{2}\sum_i (\mu_i - \psi)^2\right)
\end{aligned}
$$

MCMC

- Update each parameter with a Gibbs step
- Cycle through all parameters for each MCMC iteration

`mcmcanova.c`