STA 290 STATISTICAL LABORATORY **HOMEWORK 6**

Fall Semester 2002

Exercise (1)

Refer to the hierarchical random effects model for the fish data for modeling log(MERCURY) (log(M) as a function of log(LENGTH) log(L) and STATION (S):

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\begin{aligned}
\log(M_i) &\sim N(\alpha_{S_i} + \beta_{S_i} * (\log(L_i) - \bar{\log}(L_i)), 1/\tau) \qquad i = 1, \ldots, 171 \\
\alpha_j &\sim N(\alpha_s, 1/\tau_\alpha) \qquad j = 1, \ldots, 16 \\
\beta_j &\sim N(\beta_s, 1/\tau_\beta) \qquad j = 1, \ldots, 16 \\
\alpha_s &\sim N(0, 1/1.0E-6) \\
\beta_s &\sim N(0, 1/1.0E-6) \\
\tau &\sim Gamma(0.001, 0.001) \\
\tau_\alpha &\sim Gamma(0.1, 0.1) \\
\tau_\beta &\sim Gamma(0.1, 0.1)
\end{aligned}
$$

- Derive the conditional distribution for α_1 given all other parameters and Y.
- Derive the conditional distribution for β_1 given all other parameters and Y.
- Derive the conditional distribution for α_s given all other parameters and Y.
- Derive the conditional distribution for β_s given all other parameters and Y.
- Derive the conditional distribution for τ given all other parameters and Y.
- Derive the conditional distribution for τ_α given all other parameters and Y.
- Using BUGS (or your own program), generate samples from the predictive distributions for new locations for fish with lengths L=(15, 20, 25, 30, 35, 40, 45, 50, 60, 65, 70) cm. Turn in your code.
- Construct 95% probability intervals plotted as a function of length. Overlay prediction intervals from the classical fixed effects model with log(LENGTH) alone.
- Using the random effects model, how would your recommendations for fish sizes change compared to the fixed effects model with log(LENGTH) alone?