Problem (1)

(a) There are 26 possible letters and 10 possible digits. So the number of possible plates is $26^2 \times 10^5 = 67,600,000$.

(b) If no letter or number can be repeated, the number of possible plates is $26 \times 25 \times 10 \times 9 \times 8 \times 7 \times 6 = 19,656,000$.

Problem (7)

(a) $6! = 720$

(b) $2! \times 3! \times 3! = 72$

(c) Within the group of the boys, there are $3! = 6$ possible ways. See the group of the boys as a whole, together with the 3 girls, there are $4!$ ways. So the total is $4! \times 3! = 144$.

(d) There are two possibilities: bgbgbg or gbgbgb. Each has 3! possible orders for the boys and 3! for the girls, so the total is $2 \times 3!^2 = 72$.

Problem (9)

First choose the places for the six black blocks. Then, from the remaining 6 slots, choose the places for the 4 red blocks, then white and blue. $\binom{12}{6} \times \binom{6}{4} \times \binom{2}{1} \times \binom{1}{1} = 27720$

Problem (13)

$\binom{20}{2} = 190$

Problem (16)

(a) $\binom{6}{2} + \binom{7}{2} + \binom{4}{2} = 15 + 21 + 6 = 42$.

(b) $6 \times 4 + 7 \times 4 + 6 \times 7 = 94$

Problem (33)

(a) First invest the required minimum for each of the 4 opportunities. This uses up 11 thousand dollars, leaving 9 thousand dollars. What we need to do now is to distribute the $n = 9$ into $r = 4$ groups, with at least zero given to each. This is the same as allocating $n + r = 13$ to $r = 4$ groups, with at least one given to each (see Props 6.1, 6.2), also the same as selecting $r - 1 = 3$ places among $n - 1 = 12$ to place separating bars. Thus there are $\binom{12}{3} = 220$ possible strategies.

(b) If we don't invest in the first opportunities, then we have $\binom{14}{2}$ strategies. So the total is $\binom{13}{2} + \binom{13}{2} + \binom{14}{2} + \binom{15}{2} + \binom{12}{3}$

NOTE: Some of you worked out the problem assuming that it was not required that all of the money be invested. In this case, the problem is done by simply adding another "investment category" for the money that was held back. This solution will be fine as well.

Exercise (2)

$\sum_{i=1}^{m} n_i$

Exercise (13)

$$0 = (-1+1)^n = \sum_{i=0}^{n} \binom{n}{i} (-1)^i 1^{(n-i)} = \sum_{i=0}^{n} (-1)^i \binom{n}{i}$$

Extra Problem

There are $2^4 = 16$ possible outcomes, since each of the four facilities (two farms and two treatment plants) has two possibilities, spill and non-spill. We can represent possible outcomes by four-tuples in $S = \{S, N\}^4$, with S representing Spill and N non-spill, in alphabetical order for the four facilities (Ackerby Farms, Baker Hog Works, Cabarrus County, and Durham County), i.e.,

$$\begin{aligned} S &= \{NNNN, NNNS, NNSN, NNSS, NSNN, NSNS, NSSN, NSSS, \\ &\quad SNNN, SNNS, SNSN, SNSS, SSNN, SSNS, SSSN, SSSS\} \end{aligned}$$

where, for example, $NNSN$ represents the event that Ackerby and Baker have no spills, that Cabarrus suffers a spill, and that Durham has no spill. Certainly these sixteen elementary outcomes are not equally likely. There is no reason to expect farms and municipal water treatment plants to have identical probabilities of spillage, for example, and moreover we would expect (and hope) that non-spills are much more likely than spills, since otherwise much of North Carolina water would be polluted. The outcome $NNNN$ should have a much higher probability than any of the other fifteen outcomes, for example.