Sta 104 Homework 9 solutions

November 17, 2003

Problem 1

The are of a circle is $\pi r^2 = \pi d^2/4$. We know that $d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$, so

$$\begin{aligned} E[d] &= \int_0^1 \int_0^1 \int_0^1 \int_0^1 (\pi/4)[(x_1 - x_2)^2 + (y_1 - y_2)^2] dx_1 dx_2 dy_1 dy_2 \\ &= 2 \int_0^1 \int_0^1 (\pi/4)(x_1 - x_2)^2 dx_1 dx_2 \\ &= (\pi/2) \int_0^1 \int_0^1 [x_1^2 - 2x_1 x_2 + x_2^2] dx_1 dx_2 = \pi/12 \end{aligned}$$

Problem 2

a)

$$f(x,y) = \begin{cases} 1 & \text{if } 0 < x < 1 \text{ and } 0 < y < 2 - 2x \\ 0 & \text{otherwise} \end{cases}$$

b)

$$f(x|y=1) = \begin{cases} 2 & \text{if } 0 < x < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

so $E[x|y=1] = 1/4$.

c)

$$\begin{aligned} E[XY] &= \int_0^1 \int_0^{2-2x} xy\,dy\,dx \\ &= 1/6 \end{aligned}$$

d)

$$\begin{aligned} P[X < Y] &= \int_0^{2/3} \int_x^{2-2x} dy\,dx \\ &= \int_0^{2/3} (2 - 3x) dx \\ &= 2/3 \end{aligned}$$

e)

$$\begin{aligned}
P[Y < zX] &= \int_0^{2z/(2+z)} \int_{y/z}^{1-y/2} dx dy \\
&= \int_0^{2z/(2+z)} \left(1 - \frac{y}{2} - \frac{y}{z}\right) dy \\
&= \frac{2z}{2+z} - \left(\frac{1}{4} + \frac{1}{2z}\right) \left[\frac{2z}{2+z}\right]^2 \\
&= \frac{z}{2+z} \text{ where } 0 < z < \infty
\end{aligned}$$

Problem 3

$P = 6 \cdot \frac{1}{6^3} = \frac{1}{36}$. This is a geometric distribution, therefore the expected number of rolls is 36.

Problem 4

a) Bi(25,0.2)

b) Po(1), ASSUMING info from c); otherwise, Po(λ)

c) Bi(100,e^{-1})

d) Ge(e^{-1}) = NB(1,e^{-1})

e) Ex(λ) (λ = average # buses/hr)

f) Po($\lambda/6$) (λ = average # buses/hr)

g) Un(0,60)