Key-points for HW5

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2.12

$y|\theta \sim Poisson(\theta)$.

$$
\begin{aligned}
J(\theta) &= -E\left[\frac{d^2}{d\theta^2}log(\frac{e^{-\theta}\theta^y}{y!})\Big|\,\theta\right] \\
&= -E\left[\frac{d^2}{d\theta^2}(-\theta + ylog(\theta) - log(y!))\Big|\,\theta\right] \\
&= -E[-y\theta^{-2}|\theta] = \theta^{-1}
\end{aligned}
$$

The Jeffrey's prior :$p(\theta) \propto J(\theta)^{1/2} = \theta^{-1/2}$. The $Gamma(\alpha, \beta) \propto y^{\alpha-1}e^{-y\beta}$ then comparing to the Jeffrey's prior leads to choosing $\alpha = 1/2, \beta = 0$

2.13

a. Assume the number of fatal accidents $\sim Poisson(\theta)$, As a prior, we will use the Jefferey's prior from 2.12, $p(\theta) \propto \theta^{-1/2}$, so the posterior distribution follow as :

$$
\begin{aligned}
p(\theta|y) &\propto \theta^{-1/2}e^{-n\theta}\theta^{\sum y_i} \\
&= e^{-n\theta}\theta^{\sum y_i + 1/2}
\end{aligned}
$$

By simulation we obtain a posterior predictive interval of [14,34] in the %95 confidence level.

b. Assume a constant rate of fatal accidents with an exposure propotional to passenger miles flown. Let X_i be the miles flown, then $p(y_i|\theta) = \frac{(x_i\theta)_i^y}{y_i!}e^{-x_i\theta}$, carry out the same steps in part a), we can find the poterior and a predictive interval.

c. Carry out the same steps as above, but using Passenger Deaths instead of Fatal Accidents, we can find the posterior and the predictive interval.

d. Similar to part b).

e. As for me, the most reasonable techniques are b and d since the number of crashes and the amount of flying are not independent intuitively. For a and c, they both assuming a constant rate, that does not seem reasonable. Also, the Poisson model might not be appropriate in any case since either the fatal accidents or the passenger deaths might not be independent for every year. For example, a large number of accidents may force the improvement of safety procedure, thus reduce the accidents in the next year. We can still argue that the mean and variance of a Poisson distribution is the same. So model the fatal accidents might be more accetable.

2.18

With a vague prior, the posterior follow as $p(\theta|y) \sim N(\bar{y}, \sigma/\sqrt{n})$, so the predictive distribution follow as : $p(\tilde{y}|y) \sim N(\bar{y}, \sqrt{\sigma^2(1+1/n)})$, from this we can get the 99% predictive inteval (17.33,17.98).

2.22

a) $y|\theta \sim Exponential(\theta)$. Assume a prior on θ of $\Gamma(\alpha, \beta)$ we have one observed value, $y \geq 100$. Then

$$\begin{aligned} P(\theta|y \geq 100) &\propto \int_{100}^{\infty} p(y|\theta)\Gamma(\alpha,\beta)dy \\ &= \Gamma(\alpha,\beta)e^{-100\theta} \\ &\sim Gamma(\alpha, \beta+100) \end{aligned}$$

This has a mean of $\alpha/(\beta+100)$ and a variance of $\alpha/(\beta+100)^2$

b) If we observe $y = 100$, then our posterior becomes $Gamma(\alpha+1, \beta+100)$ which has a mean of $\frac{\alpha+1}{\beta+100}$ and a variance of $\frac{\alpha+1}{(\beta+100)^2}$

c) Although the average psterior variance would decrease, here we are averaging over $y \geq 100$. Actually$Var(\theta|y)$ is a decreasing function in y, so the variance of part a should be lower when averaging over the higher values of y. This does not contradic the identity expression.

3.3

From section 3.2, the posterior distribution for μ given a uniform prior on μ and $log(\sigma)$ is the standard Student-t distribution, $t_{n-1}(\bar{y}, s^2/n)$. For the control group, this is $t_{31}(1.013, .0018)$ and for the treatment group, it is $t_{35}(1.173, .0011)$.

By simulation, we find that the 95% posterior interval for the difference is (0.04777401 ,0.27198640). A histogram of the diferences is shown in figure 5, below.

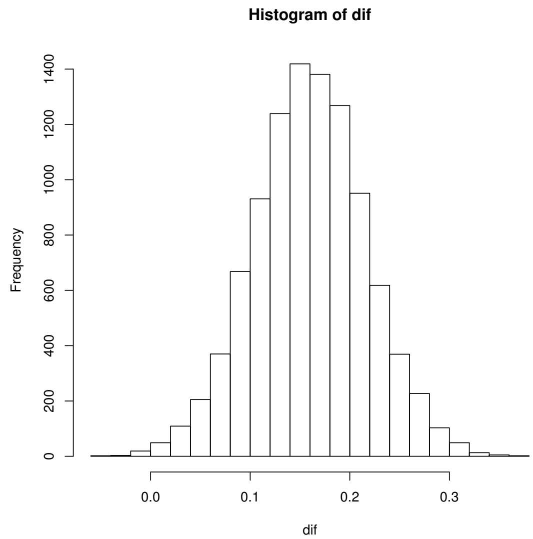

Figure 1: Empirical posterior difference of $\mu_t - \mu_c$