Key-points for HW6

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3.11

Follow the bioassay example, we get the following two figures:
Figure 1 shows that the distribution is a compromise between the likelihood and the normal prior

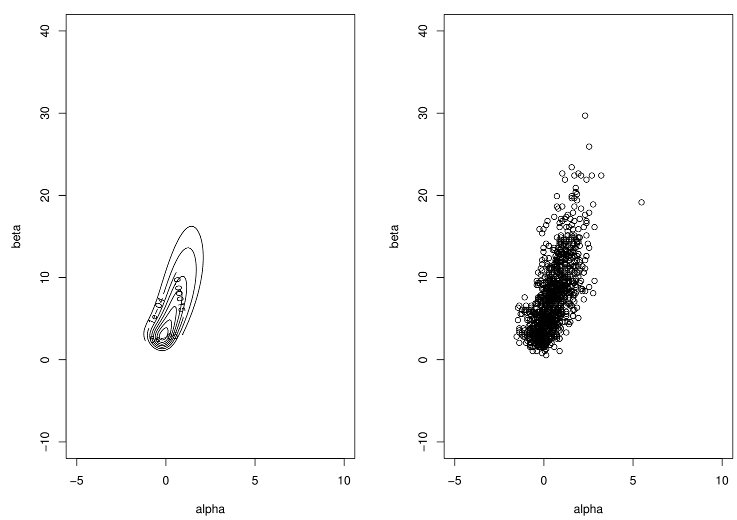

Figure 1: Contour plot and sample points

distribution.With a normal prior, the resulting posterior is going to be influenced by it. So to assume such a prior we need justify it.

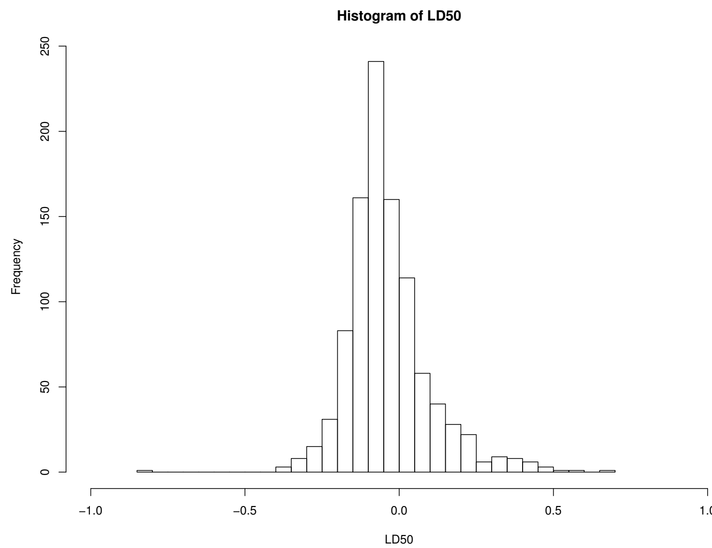

Figure 2: Ditribution of LD50

3.14

$$
\begin{aligned}
\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(\alpha,\beta|y,n,x)d\alpha d\beta &\propto p(\alpha,\beta)\Pi_{i=1}^{k} p(y_i|\alpha,\beta,n_i,x_i)d\alpha d\beta \\
&< \Pi_{i=1}^{k}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} logit^{-1}(\alpha+\beta x_i)\cdot(1-logit^{-1}(\alpha+\beta x_i))d\alpha d\beta \\
&= \Pi_{i=1}^{k}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{2+e^{\alpha+\beta x_i}+e^{-(\alpha+\beta x_i)}}d\alpha d\beta \\
&< 4\Pi_{i=1}^{k}\int_{0}^{\infty}\int_{0}^{\infty} \frac{1}{e^{\alpha+\beta x_i}+e^{-(\alpha+\beta x_i)}}d\alpha d\beta \\
&< 4\Pi_{i=1}^{k}\int_{0}^{\infty}\int_{0}^{\infty} \frac{1}{e^{\alpha+\beta x_i}}d\alpha d\beta \\
&= 4\Pi_{i=1}^{k}\frac{1}{x_i} < \infty
\end{aligned}
$$

4.2

Calculate the information matrix, and then approximate the variance by the inverse of the information matix.

4.3

Delta Method: (Refer to C&B P240) The posterior mode can be approximated by:

$$-\frac{\hat{\alpha}}{\beta} = -\frac{\hat{\alpha}}{\hat{\beta}} = -0.85/7.75 = -0.11$$

with α, β obtained by fitting a GLM. By expanding the function :$-\frac{\alpha}{\beta}$, we get the approximate variance :

$$\begin{aligned} \hat{Var}(-\frac{\alpha}{\beta}) &\approx \nabla(-\frac{\alpha}{\beta})I^{-1}(\alpha,\beta)\nabla(-\frac{\alpha}{\beta})^T|_{\hat{\alpha},\hat{\beta}} \\ &= 0.0091 \end{aligned}$$

i.e, $std \approx 0.0955$ Sampling from the normal distribution yields the following figure. Compared to the histogram in Figure 4.2, they look kind of similar .

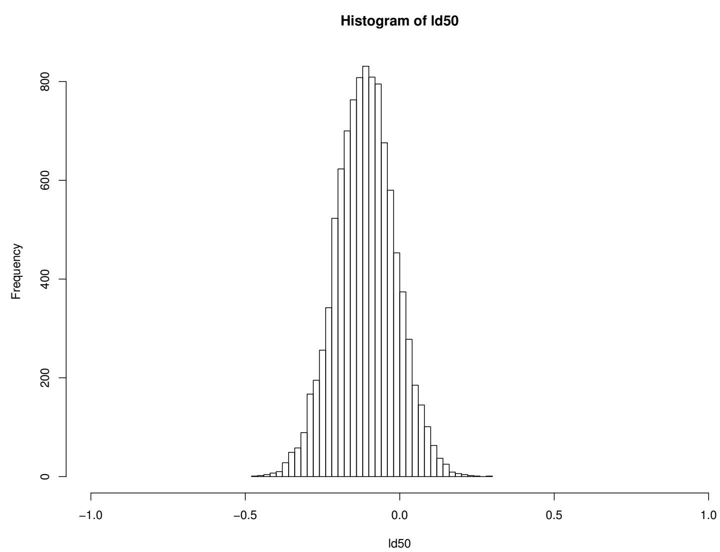

Figure 3: Ditribution of LD50 by Delta Method

11.3 Metropolis algorithm:

Arbitrarily choose the starting point to be $(\alpha_0, \beta_0, \Sigma_0) = (0.85, 7.75, (\begin{matrix} 1.09 & 3.55 \\ 3.55 & 23.74 \end{matrix}))$, which is the estimation from the GLM fitting. And take the jumping rule to be :

$$J(\theta^*|\theta^{t-1}) = N(\theta^{t-1}|\Sigma)$$

Run the Metropolis algorithm, we get the following figures:

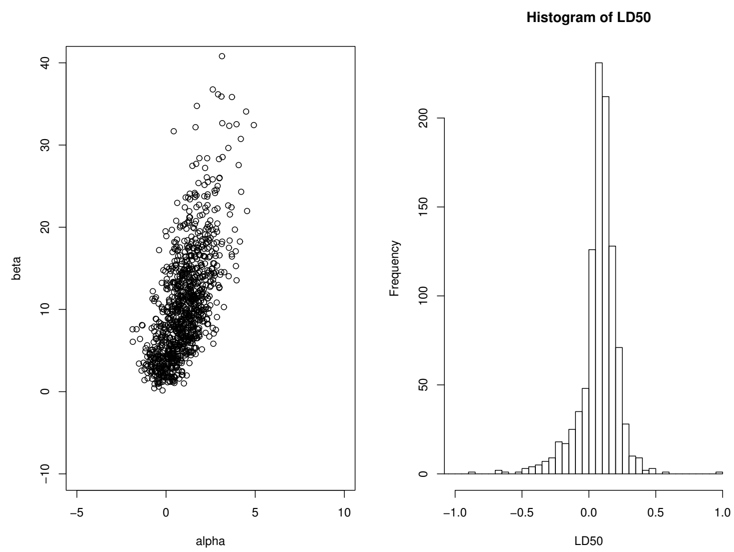

Figure 4: Ditribution of LD50 by Delta Method