21.0 Paired Differences

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21.1 Paired Differences

Example 1: You want to show that men spend less on Valentine's Day than women. You could draw some random men and some random women, ask them what they spent, and compare the means.

But maybe it is better to look at random couples, and average the difference in expenses. If the average difference (men minus women) is less than zero, this supports the alternative. If it is greater than or equal to zero, it supports the null.

Why is looking a couples a good idea?

For this example the hypotheses are:

$$\mathrm{H}_0 : \mu_m - \mu_w \geq 0 \quad \text{vs.} \quad \mathrm{H}_A : \mu_m - \mu_w < 0$$

Suppose your data are:

Man	Woman	Difference
\$20	\$30	-\$10
\$10	\$40	-\$30
\$5	\$30	-\$25
\$0	\$100	-\$100

This is a small sample ($n = 4$, not 8), and so we shall use the t_3 table. The test statistic is from part f.

The difference of means is $\bar{X}_m - \bar{X}_w = \bar{D}$, where $\bar{D}$ is the average of the differences. Thus

$$\bar{D} = \frac{1}{4}(-10 + (-30) + (-25) + (-100)) = -41.25.$$

The sd_D is the standard deviation of the differences, or

$$sd_D = \sqrt{\frac{1}{4}(\sum_{i=1}^{4} D_i^2) - \bar{D}^2} = 34.7086.$$

Thus the value of the test statistic in part f is:

$$ts = \frac{\bar{X}_m - \bar{X}_w - 0}{sd_d/\sqrt{n-1}} = \frac{-41.25}{34.7086/\sqrt{3}} = -2.05848$$

The rejection rule is III.3. We find $\mathrm{P} - value = \mathrm{P}[t_3 \leq -2.058]$, and this is between .05 and .1.

This is a bit small, but not sufficiently small that most people would reject the null hypothesis. We cannot conclude that men are more cheap than women opn Valentine's Day.

By using this paired difference test, we controlled for the duration of the relationship, intensity of passion, and probably things like socio-economic status and cultural norms.

In general, paired difference tests are good—you can draw better conclusions with fewer observations.

21.2 Significance Tests

- In a hypothesis test we either reject the null or fail to reject the null—we cannot prove the null.
- The null is like an accused person—it is presumed true until there is sufficient evidence to reject it.
- One should not data snoop; i.e., do not look at your data in order to decide which hypotheses to test.
- The fact that a test is significant does not mean that it is scientifically important—with a large enough sample size you can probably find a significant difference in anything.
- The P-value is not the probability that the null hypothesis is true.
- You need a random sample to do a significance test.