Solution for HW1

STA113 ISDS

September 5, 2004

§13.

a. $A_1 \cup A_2 = \{\text{awarded project 1 or 2}\}$

$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

$= 0.22 + 0.25 - 0.11 = 0.36$

b. $A_1' \cap A_2' =\{\text{awarded neither project 1 nor 2}\}$

$P(A_1' \cap A_2') = P((A_1 \cup A_2)') = 1 - P(A_1 \cup A_2) = 0.64$

c. $A_1 \cup A_2 \cup A_3 =\{\text{awarded at least one project from 1 ,2, and 3}\}$

$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$

$= 0.53$

d $A_1' \cap A_2' \cap A_3' =\{\text{awarded none of 1,2,3}\}$

$P(A_1' \cap A_2' \cap A_3') = P((A_1 \cup A_2 \cup A_3)') = 1 - 0.53 = 0.47$

e. $A_1' \cap A_2' \cap A_3$ ={only awarded project 3}

$P(A_1' \cap A_2' \cap A_3) = 0.17$

Hint:$P(A_1' \cap A_2' \cap A_3) + P(A_1' \cap A_2' \cap A_3') = P(A_1' \cap A_2')$

f, $(A_1' \cap A_2') \cup A_3$

={awarded project 3, or awarded neither project 1 nor 2}

$P((A_1' \cap A_2') \cup A_3) = P(A_1' \cap A_2') + P(A_3) - P(A_1' \cap A_2' \cap A_3) = 0.75$

§26.

a.$P(A_1') = 1 - P(A_1) = 0.88$

b.$P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2) = 0.06$

c.$P(A_1 \cap A_2 \cap A_3') = P(A_1 \cap A_2) - P(A_1 \cap A_2 \cap A_3) = 0.05$

d:$1 - P(A_1 \cap A_2 \cap A_3) = 0.99$

Hint: the event that the system has all of these defects and the event that the system had at most two of these defects are mutually exclusive.

§30.

a: $P_{3,8} = 336$

b: $N = \binom{30}{6} = 593775$

c: $n = \binom{8}{2}\binom{10}{2}\binom{12}{2} = 83160$

d: $P = n/N = 0.1400$

e: $\dfrac{\binom{8}{6} + \binom{10}{6} + \binom{12}{6}}{N} = 0.00196$

§33.

a: $N = \binom{25}{5} = 53,130$

b: $n = \binom{8}{4}\binom{17}{1} = 1,190$

c: $P = \dfrac{n}{N} = \dfrac{1190}{53130} = 0.022$

d: $P = \dfrac{\binom{8}{4}\binom{17}{1} + \binom{8}{5}}{N} = 0.023$

Hint: At least 4 having visible cracks means 4 or 5 of those selected having cracks

§40.

a: $n = \dfrac{P_{12,12}}{(P_{3,3})^4} = 369,600$

Hint: if the three A's,B's,C's and D s were distinguished from one another, then the answer is $n = p_{12,12} = 479,001,600$ when the subscripts are removed from A's. the number of chain molecule should be divided by $p_{3,3}$. So, the answer of this question is $n = \frac{P_{12,12}}{(P_{3,3})^4}$

b: $P(\text{All three molecules of each type end up next to one another})$
$= \frac{p_{4,4}}{n} = 6.4935e-05$
Hint:We can suppose the three molecules of the same type to be 1 molecule,(such as BBBAAADDDCCC=BADC), then the number of the chain molecule described in the question is $P_{4,4}$

§48.

a: $P(A_2|A_1) = \frac{P(A_1 \cap A_2)}{P(A_1)} = \frac{0.06}{0.12} = 0.5$

b: $P(A_1 \cap A_2 \cap A_3|A_1) = \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1)} = \frac{1}{12}$

c: $P((A_1 \cap A_2' \cap A_3') \cup (A_1' \cap A_2 \cap A_3') \cup (A_1' \cap A_2' \cap A_3)|A_1 \cup A_2 \cup A_3)$
$= \frac{P((A_1 \cap A_2' \cap A_3')) + P((A_1' \cap A_2 \cap A_3')) + P((A_1' \cap A_2' \cap A_3))}{P(A_1 \cup A_2 \cup A_3)}$
$= 0.05/0.14 = 0.3571$

d: $P(A_3'|A_1 \cap A_2) = \frac{P(A_3' \cap A_1 \cap A_2)}{P(A_1 \cap A_2)} = 0.05/0.06 = 0.8333$

Hint:Draw a venn diagram

§59.

a: $P(A_2 \cap B) = P(B|A_2)P(A_2) = 0.21$

b: $P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3) = 0.455$

c: $$P(A_1|B) = \frac{P(B|A_1)P(A_1)}{\sum_{i=1}^{3} P(B|A_i)P(A_i)} = 0.2637$$
$$P(A_2|B) = \frac{P(B|A_2)P(A_2)}{\sum_{i=1}^{3} P(B|A_i)P(A_i)} = 0.4615$$
$$P(A_3|B) = \frac{P(B|A_3)P(A_3)}{\sum_{i=1}^{3} P(B|A_i)P(A_i)} = 0.2747$$

§61.

Let $A_i =$ {i defective components in the batch}, for $i = 0, 1, 2$

$B_a =$ {neither tested component is defective}

$B_b =$ {One of the tested components is defective}

a: $$P(A_0|B_a) = \frac{1 \times 0.5}{1 \times 0.5 + \binom{9}{2}/\binom{10}{2} \times 0.3 + \binom{8}{2}/\binom{10}{2} \times 0.2}$$
$$= \frac{1 \times 0.5}{1 \times 0.5 + 0.8 \times 0.3 + 0.622 \times 0.2} = 0.5784$$

$P(A_1|B_a) = 0.2776$

$P(A_2|B_a) = 0.1439$

Hint:for $i = 0, 1, 2$

$$P(A_i|B_a) = \frac{P(B_a|A_i)P(A_i)}{\sum_{j=0}^{2} P(B_a|A_j)P(A_j)}$$

b: $P(A_0|B_b) = 0$

$$P(A_1|B_b) = \frac{\binom{9}{1}/\binom{10}{2} \times 0.3}{\binom{9}{1}/\binom{10}{2} \times 0.3 + \binom{8}{1}\binom{2}{1}/\binom{10}{2} \times 0.2}$$

$$= \frac{0.2 \times 0.3}{0.2 \times 0.3 + 0.3556 \times 0.2} = 0.4576$$

$P(A_2|B_b) = 0.5424$

Hint:for $i = 0, 1, 2$

$$P(A_i|B_b) = \frac{P(B_b|A_i)P(A_i)}{\sum_{j=0}^{2} P(B_b|A_j)P(A_j)}$$

§69.

a: $P(B^{'}|A^{'}) = P(B^{'}) = 0.3$

Hint: A and B are independent events

b: $P(A \cup B) = P(A) + P(B) - P(A)P(B) = 0.82$

c: $$P(A \cap B^{'}|A \cup B) = \frac{P((A \cap B^{'}) \cap (A \cup B))}{P(A \cup B)}$$

$$= \frac{P(A \cap B^{'})}{P(A \cup B)} = \frac{0.12}{0.82} = 0.1463$$

§78.

let event S_i ={subsystem i works} for $i = 1, 2.$ and C_i ={component i works} for $i = 1, 2, 3, 4$

P(system works)$= 1 - P(S_1^{'} \cap S_2^{'}) = 1 - P(S_1^{'})P(S_2^{'})$

$= 1 - P(C_1^{'} \cap C_2^{'})P(1 - C_3 \cap C_4) = 1 - 0.1^2 \times (1 - 0.9^2) = 0.9981$

Hint: $S_1^{'}$ and $S_2^{'}$ are independent events.

§80.

Notice $P(A) = 1/6,\ P(B) = 1/6,\ P(c) = 1/6$

a: A and B are independent events;

b: B and C are independent events;

Proof: $P(B \cap C) = P(A \cap B) = \frac{1}{36} = P(B)P(C)$

c: A and C are independent events;

Proof: $P(A \cap C) = P(A \cap B) = \frac{1}{36} = P(A)P(C)$

d: A,B and C aren't independent events;

Proof:$P(A \cap B \cap C) = \frac{1}{36} \neq \frac{1}{216}$=P(A)P(B)P(C)