- 1(a) $p(1,1) = .20$

  (b) $p(0,0) + p(0,1) + p(1,0) + p(1,1) = .42$

  (c) At least one hose is in use at both islands. $p(1,1) + p(1,2) + p(2,1) + p(2,2) = .70$

  (d) $p_X(x) = .16, .34, .50$ for $x = 0, 1, 2$.

  $p_Y(y) = .24, .38, .38$ for $y = 0, 1, 2$.

  $P(X \leq 1) = 0.50$

  (e) $p(0,0) = .10 \neq p_X(0) \cdot p_Y(0)$. So X and Y are NOT independent.

- 2(a) $p(x,y) = p_X(x) \cdot p_Y(y)$ for $x = 0, 1, 2$ and $y = 0, 1, 2, 3, 4$.

  (b) .56

  (c) .30

  (d) .53

- 6(c) Use the hint for (a)

$$\begin{aligned} P(X &= x, Y = y) = P(X = x) \cdot P(Y = y | X = x) \\ &= p_X(x) \binom{x}{y} (.6)^y (.4)^{x-y} \end{aligned}$$

  for $x = 0, 1, 2, 3, 4$ and $0 \leq y \leq x$

  The joint pmf is given by

	Y=0	Y=1	Y=2	Y=3	Y=4
X=0	.1	0	0	0	0
X=1	.08	.12	0	0	0
X=2	.048	.144	.108	0	0
X=3	.016	.072	.108	.054	0
X=4	.004	.0230	.052	.052	.019

  The marginal pmf of Y is the column sum.

  (a) .052

  (b) $p(0,0) + p(1,1) + p(2,2) + p(3,3) + p(4,4) = .401$

- 8(a)

$$p(3,2) = \frac{\binom{8}{3}\binom{10}{2}\binom{12}{1}}{\binom{30}{6}} = .0509$$

  (b)

$$p(x,y) = \frac{\binom{8}{x}\binom{10}{y}\binom{12}{6-x-y}}{\binom{30}{6}}$$

  for $0 \leq x, y \leq 6$ and $x + y \leq 6$.

- 9(a) = 3/380,000

  (b) $\int_{20}^{26}\int_{20}^{26} K(x^2+y^2)dxdy = .3024$

  (d) $f_X(x) = 10Kx^2 + .05$ for $20 \le x \le 30$.

  (e) NOT independent.

- 13(a) $f(x,y) = e^{-x-y}$ for $x, y \ge 0$

  (b)=.400

- 15(a) Let event $A = (X_1 \le y)$ and event $B = (X_2 \le y) \cap (X_3 \le y)$.

$$\begin{aligned} F(y) &= P(Y \le y) = P[\ A \cup B\ ] \\ &= P(A) - P(B) - P(A \cap B) \\ &= (1-e^{-\lambda y}) - (1-e^{-\lambda y})^2 - (1-e^{-\lambda y})^3 \quad \text{for } y \ge 0 \end{aligned}$$

  So the pdf is equal to

$$f(y) = F'(y) = 4\lambda e^{-2\lambda y} - 3\lambda e^{-3\lambda y}, \quad \text{for } y \ge 0$$

- (b) $E(Y) = \int_0^\infty y \cdot f(y)dy = \frac{2}{3\lambda}$.

- 18(a) .235, .588, .176

  (b) .12, .28, .60

  (c) .40

  (d) .05326 .1579, .7895

- 19(a)

$$\begin{aligned} f_{Y|X}\ (y \mid x) &= \frac{k(x^2+y^2)}{10kx^2+.05}, \quad 20 \le y \le 30 \\ f_{X|Y}\ (x \mid y) &= \frac{k(x^2+y^2)}{10ky^2+.05}, \quad 20 \le x \le 30 \end{aligned}$$

  (b) $P(Y \ge 25 \mid X = 22) = \int_{25}^{30} f_{Y|X}\ (y \mid 22)\ dy = .556$

  $P(Y \ge 25) = .549$

  (c) $E(Y \mid X = 22) = 25.3729$

  $E(Y^2 \mid X = 22) = 652.0286$

  $V(Y \mid X = 22) = 8.2440$

  Standard deviation = $\sqrt{8.2440}$

- 30(a) $E(XY) = 44.25, E(X) = 5.55, E(Y) = 8.55$. Cov(X,Y)= -3.2025
  (b) $\rho = -0.2074$
- 31(a) $E(X) = E(Y) = 25.329$
  $E(XY) = 641.447, \;\; Cov(X, Y) = -.111$
  (b) $Var(X) = Var(Y) = 8.264$
  $\rho = -.0134$