STA 113 Homework

Chapter 8

November 8, 2004

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a At significance level $\alpha = 0.01$
1:Parameter of interest: $\mu =$ true average percentage of SiO_2.
2:Null hypothesis: $H_0 : \mu = 5.5$(null value = 5.5)
3:Alterntive hypothesis: $H_\alpha : \mu \neq 5.5$
4:Test statistic value

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{\bar{x} - 5.5}{0.3/\sqrt{n}} = -0.33$$

5: Rejection region : The form of H_α implies use of a two- tailed test with rejection when $z \leq -z_{.005}$ or $z \geq z_{.005}$. $z_{0.005} = 2.58$, So we reject H_0 if $z \leq -2.58$ or $z \geq 2.58$
6:Plug in $n = 16$ and $\bar{x} = 5.25$,

$$z = \frac{5.25 - 5.5}{0.3/\sqrt{16}} = -3.33$$

7: The computed value of z falls in the rejection region $(-\infty, -2.58)$, so H_0 is rejected at significance level 0.01.It means that the data gives strong support to the claim that the true average differs from 5.5.

b $1 - \beta(5.6) = 1 - \Phi(2.58 + \frac{5.5 - 5.6}{0.3/\sqrt{16}}) + \Phi(-2.58 + \frac{5.5 - 5.6}{0.3/\sqrt{16}})$
$= -.105$

c The sample size n should be:
$n = [\frac{\sigma(z_{0.005} + z_{0.01})}{\mu_0 - \mu}]^2 = [\frac{0.3 \cdot (-2.58 - 1.28)}{0.1}]^2 \approx 217$

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a The alternative of interest here is $H_\alpha : p > .5$, so the rejection region should consist of large values of X. So we should use an upper-tailed test.Thus, the first rejection region is most appropriate.

b

$$
\begin{aligned}
p(\text{type I error}) &= p(reject H_0 | p = 0.5) \\
&= p(X \geq 15 \text{ where} X \sim \text{Bin}(20, 0.5)) \\
&= 1 - p(X \leq 14 \text{ where} X \sim \text{Bin}(20, 0.5)) \\
&= 1 - B(14, 20, 0.5) \\
&= 0.02069 \leq 0.05
\end{aligned}
$$

So, this region specify a level .05 test
For the region of $14, 15, \cdots, 20$, $\alpha = 0.058 > .05$, so this region does not specify a level .05 test. So the chosen region of part(a) is the best level .05 test.

c

$$
\begin{aligned}
\beta(0.6) &= p(\text{ can't reject} H_0 | p = 0.6) \\
&= p(x < 15 \text{ where} X \sim Bin(20, 0.6)) \\
&= 0.8744
\end{aligned}
$$

$\beta(0.8) = B(14, n = 20, p = 0.8) = 0.1958$

d The best level .10 test is specified by the rejection region $14, 15, \cdots, 20$ (with $\alpha = 0.052$). Since 13 is not in the rejection region, H_0 is not rejected at this level.

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a $H_0 : \mu = 2150$ and $H_\alpha : \mu > 2150$.

b We can use one sample t Test
The test statistic is

$$T = \frac{\bar{x} - \mu}{S/\sqrt{n}}$$

c For this data, plug in $\bar{x} = 2160$,$\mu = 2150$ and $S = 30$, $n = 16$
$T = \dfrac{2160 - 2150}{30/\sqrt{16}} = 1.333$

d In this case, it is Upper-tailed t test. Assuming H_0 is true,T has a t distribution with n-1 degrees of freedom
$p = p(T > 1.333) = 0.1012$

d For a level 0.05 test, H_0 cannot be rejected.That data doesn't give strong support to the claim that the true average is larger than 2150

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a $H_0 : p = 1/75$ and $H_\alpha : p \neq 1/75$.
We reject H_0 if either $z \geq 1.96$ or $z \leq -1.96$.
The estimate value for p is $\hat{p} = \frac{16}{800} = .02$,
In this data,
$$z = \frac{.02 - .01333}{\sqrt{\frac{.01333 \cdot .98667}{800}}} = 1.645$$
Which is not in either rejection region.Thus, we fail to reject the null hypothesis. There is not evidence that the incidence rate among prisoners differs from that of the adult population. The possible errors we could have made in type II error.

b P-value $= 2[1 - \Phi(1.645)] = 2.05 = .10$.
Since $.10 < .20$,we could reject H_0.