

STA 114: STATISTICS

HW 1

Due Wed Sep 7 2011

This homework is intended to be a review of the materials you are expected to be familiar with. You may consult chapters 1-5 of DeGroot and Schervish for a brush up.

1. Suppose a researcher thinks that a 10% of all human beings are left-handed. What's the probability that in a randomly chosen sample of 10 students she'd have at least one left hander? What's the probability that in a randomly chosen sample of 500 students she's have at least 50 left handers?
2. Let  $U \sim \text{Uniform}(0, 1)$ . Find the pdf of the random variable  $X = \log \frac{U}{1-U}$ .
3. Let  $X$  have the exponential distribution with mean 1. Find  $EY$  where  $Y = \min(X, 1)$ .
4. Let  $N \sim \text{Poisson}(\lambda)$  and  $Y$ , given  $N = n$ , be conditionally distributed as  $\text{Binomial}(n, p)$ , where  $\lambda > 0$  and  $p \in (0, 1)$ . Show that the marginal distribution of  $Y$  is  $\text{Poisson}(\lambda p)$ .
5. Let  $X$  and  $Y$  be independent  $\text{Exponential}(1)$  random variables. Define  $V = X + Y$  and  $W = X - Y$ .
  - (a) Find the joint density of  $V$  and  $W$ .
  - (b) Find the marginal densities of  $V$  and  $W$ .
  - (c) Are  $V$  and  $W$  independent? Justify your answer.
6. Suppose  $U_1, U_2, \dots, U_{100}$  are independent  $\text{Uniform}(0, 1)$  random variables. Find the probability that  $U_1 + U_2 + \dots + U_{100} > 55$ .