STA 114: Statistics

HW 11

Due Wed Dec 7 2011

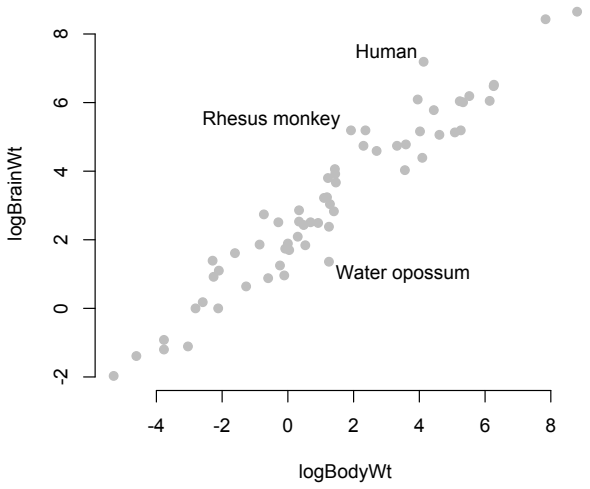
1. Consider the linear model $Y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, $\epsilon_i \overset{\text{IID}}{\sim} \mathsf{Normal}(0, \sigma^2)$ where (x_i, Y_i), $i = 1, \cdots, n$ are data on $X =$ the boiling point of water (in degrees F) and $Y =$ atmospheric pressure (in inches Mercury). For observed data we have

$$n = 17, \;\; \bar{x} = 202.95, \;\; s_x^2 = 33.18, \;\; \bar{y} = 25.06, \;\; s_{xy} = 17.35, \;\; s_{y|x}^2 = 0.0542.$$

Answer the following based on these observations.

   (a) Write down the equation of the least squares line.
   (b) Report 90%, 95% and 99% ML confidence intervals for each of β_0 and β_1.
   (c) Report p-value for $H_0 : \beta_1 = 0.5$ against $H_1 : \beta_1 \neq 0.5$ based on ML tests.
   (d) Report p-value for $H_0 : \beta_1 \leq 0.5$ against $H_1 : \beta_1 > 0.5$ based on ML tests.
   (e) Report a 95% predictive interval for atmospheric pressure when the water boiling point is recorded to 190 degrees F.
   (f) Atmospheric pressure at altitude h (in thousand feet) above the sea level equals $29.92(1 - 0.074401 \cdot h)^{5.25588}$ inches Mercury. Give a 95% predictive interval for altitude when the water boiling point is measured to be 190 degrees F.

2. The figure below shows log brain weight (in log-grams) against log body weight (in log kilograms) for 62 mammals (three of them are marked). The actual measurements are given below. Remember that these are paired observations, so the first numbers from the two cell (1.22, 3.8) give the brain and body weight (in logarithm) of a single mammal, and so on.



Log Brain weight	1.22, -0.73, 0.3, 6.14, 3.59, 3.32, 2.7, 0.04, 1.43, -0.86, -2.29, -0.08, 0, -5.3, -2.81, 1.25, 0.69, 0.53, 7.84, -3.77, 5.23, 6.26, -0.24, 2.3, 1.19, -1.61, 0.34, 6.27, 5.33, 4.44, -0.29, 4.13, 8.8, 1.25, 1.92, 3.56, 1.4, -2.12, -3.77, -4.61, 0.34, 5.52, 0.92, 4.02, 4.61, 3.95, 2.36, -0.6, 4.09, 1.28, 1.46, -1.27, -2.59, -2.1, -3.04, 5.26, 1.1, 5.08, -0.11, 0.48, -2.26, 1.44
Log Body weight	3.8, 2.74, 2.09, 6.05, 4.78, 4.74, 4.59, 1.7, 4.06, 1.86, 1.39, 1.74, 1.89, -1.97, 0, 2.38, 2.51, 1.84, 8.43, -1.2, 6.04, 6.48, 1.25, 4.74, 3.24, 1.61, 2.86, 6.52, 6.01, 5.78, 2.51, 7.19, 8.65, 1.36, 5.19, 4.03, 2.83, 0, -0.92, -1.39, 2.53, 6.19, 2.49, 5.16, 5.06, 6.09, 5.19, 0.88, 4.39, 3.04, 3.67, 0.64, 0.18, 1.1, -1.11, 5.19, 3.22, 5.13, 0.96, 2.43, 0.92, 3.92

Test how well does the simple linear regression model, with log body weight as the explanatory variable and log brain weight as the response, fit the observed measurements? Follow the binning protocols we have discussed before for implementing Pearson's chi-square goodness-of-fit test.