STA 114: Statistics

HW 4

Due Wed Sep 28

1. A set of n lactic acid measurements are modeled as $X_1, \cdots, X_n \overset{\text{IID}}{\sim} \mathsf{Normal}(\mu, \sigma^2)$, $(\mu, \sigma^2) \in (-\infty, \infty) \times (0, \infty)$. Write down ML 90%, 95% and 99%-CIs for μ based on data (0.86, 1.53, 1.57, 1.81, 0.99, 1.09, 1.29, 1.78, 1.29, 1.58).

2. In a survey of $n = 500$ randomly chosen college students, 200 students said they support a certain federal policy. Assuming a $p \in [0, 1]$ fraction of all students in the college support the policy, give ML, asymptotically 90%, 95% and 99% CIs for p.

3. Annual hurricane counts from a set of n consecutive years are modeled as $X_1, \cdots, X_n \overset{\text{IID}}{\sim} \mathsf{Poisson}(\mu)$, $\mu \in (0, \infty)$. Give ML, asymptotically 90%, 95%, 99% CIs for μ based on data (12, 14, 15, 12, 16, 14, 27, 10, 14, 16).

4. A set of n time intervals (in minutes) between two successive eruptions of a geyser is modeled as $X_1, \cdots, X_n \overset{\text{IID}}{\sim} \mathsf{Exponential}(\lambda)$, $\lambda \in (0, \infty)$. Give ML, asymptotically 90%, 95%, 99% CIs for λ based on data (79, 54, 74, 62, 85, 55, 88, 85, 51, 85).

5. Consider the model $X_1, \cdots, X_n \overset{\text{IID}}{\sim} \mathsf{Normal}(\mu, \sigma^2)$, $(\mu, \sigma^2) \in (-\infty, \infty) \times (0, \infty)$. Give the expression of a 95% CI for $\eta = \exp(\mu)$.

6. For the model $X_1, \cdots, X_n \overset{\text{IID}}{\sim} \mathsf{Exponential}(\lambda)$, $\lambda \in (0, \infty)$, prove that $[\bar{x}^{-1}(1 - \frac{z(\alpha)}{\sqrt{n}}), \bar{x}^{-1}(1 + \frac{z(\alpha)}{\sqrt{n}})]$ is an asymptotically $100(1-\alpha)$%-CI for λ [Hint: use CLT for $\bar{X}$.]

7. Consider the model $X_1, \cdots, X_n \overset{\text{IID}}{\sim} \mathsf{Normal}(\mu, \sigma^2)$, $(\mu, \sigma^2) \in (-\infty, \infty) \times (0, \infty)$. It is known that if $X_i \overset{\text{IID}}{\sim} \mathsf{Normal}(\mu, \sigma^2)$ then $\frac{\bar{X} - \mu}{s_X / \sqrt{n}}$ is asymptotically distributed as $\mathsf{Normal}(0, 1)$, and hence $\bar{x} \mp z(\alpha) s_x / \sqrt{n}$ is an asymptotically $100(1-\alpha)$%-CI for μ. Find the exact confidence coefficient for this interval for each of $\alpha = 0.1, 0.05, 0.01$ and each of $n = 10, 20, 100$ (Report numbers, not expressions, but back them by showing your calculations. You can use t-tables, z-tables, or corresponding `R` functions).

8. Consider two observations $X_1, X_2 \overset{\text{IID}}{\sim} g(x_i|\theta)$, $\theta \in (-\infty, \infty)$, where $g(y|\theta)$ is the following pmf: $g(y|\theta) = 0.5$ if $y = \theta + 1$ or $y = \theta - 1$, and $g(y|\theta) = 0$ for all other y. Consider the following "interval" procedure $A(x)$ for θ based on data $x = (x_1, x_2)$:

$$A(x) \text{ is the singleton set containing the point } \begin{cases} \frac{x_1 + x_2}{2} & \text{if } x_1 \neq x_2 \\ x_1 - 1 & \text{if } x_1 = x_2 \end{cases}.$$

   (a) Compute the confidence coefficient of A.

   (b) If your observe $x = (1, 3)$, what is your best guess for θ? How certain are you? Explain.