Composite Hypotheses
Example 1

- $H_0: X_i \sim N(\mu_0, \sigma^2)$
- $H_1: X_i \sim N(\mu_1, \sigma^2)$,

$\sigma^2$ unknown.

Example 2

- $H_0: X_i \sim Bin(n = 100, p = 0.5)$
- $H_1: X_i \sim Bin(n = 100, p < 0.5)$

Two-sided (or two-tailed) $p \neq 0.5$ versus one-sided $p < 0.5$.

Usually NO TEST will be UNIFORMLY most powerful against ALL alternatives, except Monotone Likelihood Ratio.

Monotone Likelihood Ratio
If $H_0$ and $H_1$ assert that theta lies in $\Theta_0$ and $\Theta_1$, respectively, and if

$$f(x|\theta_1)/f(x|\theta_0)$$

is a monotonic increasing function of $T(x)$ for every $\theta_1$ in $\Theta_1$ and every $\theta_0$ in $\Theta_0$, then

Reject if $T(x) > c$

is Most Powerful test *uniformly* in $\Theta_0$ and $\Theta_1$, or “UMP”.

Generalized Likelihood Ratio Tests
One approach in problems with composite hypotheses is to base tests on the “generalized likelihood ratio” statistic:

$$\frac{\sup_{\theta \in \Theta_1} f(x|\theta)}{\sup_{\theta \in \Theta_0} f(x|\theta)}$$

This is an extension of the standard likelihood ratio, only we find the MLE within each hypothesis at the numerator and denominator.

Examples: Normal distribution

- Known variance: Z-test
- Unknown variance: t-test
Bayesian testing
Return to Poisson example.

- $H_0$: $X_i \sim \text{Poisson}(2)$, prior $p_0$
- $H_1$: $X_i \sim \text{Poisson}(3)$, prior $p_1$

\[
p(H_0|X_1, \ldots, X_n) = \frac{p_0 f(X|H_0)}{p_0 f(X|H_0) + p_1 f(X|H_1)}
= \frac{1}{1 + \frac{p_1}{p_0} \Lambda_{H_1, H_0}}
\]