STA 611: Introduction to Statistical Methods – Fall 2012

Homework 10 – due November 29, 2012
Part I

Part II will be posted on Tuesday, November 20

These exercises are meant to be representative of the material in Chapters 9.1 and 9.5-9.7 in DeGroot and Schervish.

1. A scientist desperate for publication is tempted to choose the significance level α_0 *after* seeing the data in such a way that the H_0 is rejected (or not rejected). In this problem you will see what the true probability of Type I and Type II errors is in this practice. For the following two trivial test procedures, give the power function and the probabilities of Type I and Type I errors.

   (a) Test procedure δ_r: Always reject H_0 no matter what data are obtained (equivalent to the practice of choosing the α_0 level after seeing data to force rejection of H_0)

   (b) Test procedure δ_a: Never reject H_0 no matter what data are obtained (equivalent to the practice of choosing the α_0 level after seeing data to force not rejecting H_0)

   Do you still think it is OK to choose the significance level α_0 after seeing data?

2. Show that the two sided t-test is a Likelihood ratio test (see equation 9.5.7 on page 582).

3. A courier service in Reykjavík advertises that its average delivery time is less than 6 hours for deliveries within the Reykjavík metropolitan area (that includes the towns Kópavogur, Garðabær, Hafnarfjörður, Mosfellsbær and Seltjarnarnes). A random sample of 12 deliveries were selected and the delivery time (in hours) was recorded:

   $$3.03, 6.33, 6.50, 5.22, 3.56, 6.76, 7.98, 4.82, 7.96, 4.45, 5.09, 6.46$$

   Is this sufficient evidence to support the courier's advertisement, at the 5% significance level?

4. Let $X_1, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$ where both μ and σ^2 are unknown. Say we are interested in testing the hypotheses

   $$\sigma^2 \geq \sigma_0^2 \qquad \text{vs.} \qquad \sigma^2 < \sigma_0^2$$

for a given positive number σ_0^2. Let δ be the test procedure that rejects H_0 if $S_n^2/\sigma_0^2 \leq c$ where

$$S_n^2 = \sum_{i=1}^{n}(X_i - \overline{X}_n)^2 .$$

(a) Determine the power function for δ

(b) Find c so that δ is a level α_0 test

(c) For observed $S_n^2 = s_n^2$ give a formula for the p-value.

(d) Use this test to test the hypotheses

$$\sigma^2 \geq 2.5 \qquad \text{vs.} \qquad \sigma^2 < 2.5$$

for the Reykjavík courier service data in the previous exercise, at the 5% significance level.

5. Let $X_1, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$ where both μ and σ^2 are unknown. Using the $100\gamma\%$ two-sided (symmetric) confidence interval for σ^2 (see e.g. exercise 6 on Homework 8) give a $\alpha_0 = 1 - \gamma$ level test procedure for testing the hypotheses

$$\sigma^2 = \sigma_0^2 \qquad \text{vs} \qquad \sigma^2 \neq \sigma_0^2$$

6. Let $X_1, \ldots, X_m$ be a random sample from $N(\mu_1, \sigma^2)$ and $Y_1, \ldots, Y_n$ be a random sample from $N(\mu_2, \sigma^2)$ where μ_1, μ_2 and σ^2 are unknown. Suppose we are interested in testing the hypotheses

$$H_0 : \mu_1 \geq \mu_2 \qquad \text{vs.} \qquad H_1 : \mu_1 < \mu_2$$

and let

$$S_X^2 = \sum_{i-1}^{m}(X_i - \overline{X}_m)^2 \quad \text{and} \quad S_Y^2 = \sum_{i-1}^{n}(Y_i - \overline{Y}_n)^2$$

$$U = \frac{\sqrt{m+n-2}\,(\overline{X}_m - \overline{Y}_n)}{\left(\frac{1}{m} + \frac{1}{n}\right)^{1/2}\left(S_X^2 + S_Y^2\right)^{1/2}}$$

Let δ be the test procedure that rejects H_0 if $U \leq T_{m+n-2}^{-1}(\alpha_0)$.

(a) Give the power function of δ and show that δ is a size α_0 test.

(b) For $U = u$, show that the p-value is $T_{m+2-2}(u)$

7. Samples of wood were obtained from the core and periphery of a Byzantine church. The age of the wood was determined, giving the following data

Core				Periphery		
1294	1251	1279	1248	1284	1274	1272
1274	1240	1264	1232	1264	1256	1256
1263	1220	1254	1218	1254	1250	1242
1251	1210					

You can assume that all the measurements are independent, that the core data are from the same normal distribution and that the periphery data are from another normal distribution.

(a) Test the hypothesis that the variances in these two normal distributions are equal.

(b) Use the two-sample t-test to determine if the mean age of the core is the same as the mean age of the periphery. For this test, is it reasonable to assume that the variances are equal?