STA 611: Introduction to Statistical Methods – Fall 2012

Homework 10 – due November 29, 2012
Part II

These exercises are meant to be representative of the material in Chapters 9.8-9.9 in DeGroot and Schervish.

In both questions 8. and 9. assume you are operating under the generalized 0-1 loss:

$L(\theta, d_j)$	d_0	d_1
$\theta \in \Omega_0$	0	w_0
$\theta \in \Omega_1$	w_1	0

where d_0 is the decision to not reject $H_0 : \theta \in \Omega_0$ and d_1 is the decision to reject H_0.

8. Consider again the measurements you saw in homework 8, problem 6 and homework 9, problem 2:

$$0.95, 0.85, 0.92, 0.95, 0.93, 0.86, 1.00, 0.92, 0.85, 0.81$$
$$0.78, 0.93, 0.93, 1.05, 0.93, 1.06, 1.06, 0.96, 0.81, 0.96$$

Again, assume that these data are observations of independent $N(\mu, 1/\tau)$ random variables where μ and τ are unknown. Assume that $w_1 = 0.05$ and $w_0 = 0.95$.

(a) Let μ_1, λ_1, α_1 and β_1 be the parameters of the Normal-Gamma *posterior* distribution of (μ, τ). Give the Bayes test procedure for testing the hypotheses

$$H_0 : |\mu - \mu_0| \leq d \quad \text{vs. } H_1 : |\mu - \mu_0| > d \tag{1}$$

(b) As before, assume the Normal-Gamma prior distribution for (μ, τ) with parameters $\mu_0 = 1$, $\lambda_0 = 1.5$, $\alpha_0 = 9$ and $\beta_0 = 3$. Use the Bayes test procedure to test the hypotheses

$$H_0 : |\mu - 1| \leq 0.05 \quad \text{vs. } H_1 : |\mu - 1| > 0.05 \tag{2}$$

(c) Now assume the improper prior $p(\mu, \tau) = 1/\tau$. Use the Bayes test procedure to test the hypotheses in (2).

9. Ragna Ingólfsdóttir is the best female badminton player in Iceland these days (competed in both the 2008 and 2012 Summer Olympics). Tinna Helgadóttir is another great player although she has not made it to the Olympics (yet). The number of shuttlecocks used in a match between Ragna and Tinna has a Poisson distribution with mean parameter λ. Two badminton enthusiasts and former champions Broddi and Elsa disagree about λ. Broddi thinks it's $\lambda = 4$, while Elsa thinks it's $\lambda = 5$. Unfortunately they only have one data point, in their last match Ragna and Tinna used $x = 7$ shuttlecocks.

   (a) Broddi and Elsa are Bayesians and have equal prior probabilities $p_0 = p_1 = 0.5$ for their favorite value. They want to test the hypotheses

   $$H_0 : \lambda = 4 \quad \text{v.s.} \quad H_1 : \lambda = 5$$

      i. What is the posterior probability that Broddi is right?
      ii. Given that $w_1 = 0.1$ and $w_0 = 0.9$ will the Bayes test procedure reject H_0?
      iii. If Broddi and Elsa had instead tested the hypotheses

      $$H_0 : \lambda = 5 \quad \text{v.s.} \quad H_1 : \lambda = 4$$

      would the Bayes test procedure make the same decision about λ as in i.? (Hint: you need to make sure that the loss function you use here is consistent with the one you used in i.)

   (b) Suppose now that Broddi and Elsa agree that any value of λ is possible and now they want to test the hypotheses

   $$H_0 : \lambda \leq 4.5 \quad \text{v.s.} \quad H_1 : \lambda > 4.5$$

   They agree on a prior distribution for λ: The Gamma$(9, 2)$ distribution, which has mean $9/2 = 4.5$

      i. Find the posterior distribution for λ given $x = 7$
      ii. Give a sum or integral expression for calculating the probability that Broddi is closer to the right answer, i.e. $P(\lambda \leq 4.5 | x = 7)$. No need to give the exact numerical answer of 0.2822072 but be explicit and simplify.
      iii. Given that $w_1 = 0.1$ and $w_0 = 0.9$ will the Bayes test procedure reject H_0?