STA 611: Introduction to Statistical Methods – Fall 2012
Math Quiz Solutions

Familiarity to the following concepts is essential for your success in this course:

Function, limit, continuity, differentiation (e.g. product rule, quotient rule and chain rule), integration, integration techniques such as integration by parts and by substitution, function of two variables, double integrals, infinite series, maxima and minima of functions.

Take this quiz by yourself, preferably before classes start and no later than by our second class (Thursday August 30) to help you determine whether you have the necessary mathematical background. It should take no more than an hour. Do not look at the solutions beforehand!

1. Express $9x^2 + 12x + 10$ in the form $a(x + b)^2 + c$, where a, b and c are not functions of x.

$$9x^2 + 12x + 10 = (3x + 2)^2 + 6 = 9(x + 2/3)^2 + 6$$

2. Express the following expressions in terms of $\log(x)$ and $\log(y)$:

   (a) $\log(x^3) = 3\log(x)$

   (b) $\log(1/x^2) = -2\log(x)$

   (c) $\log(x/y) = \log(x) - \log(y)$

   (d) $\log(xy) = \log(x) + \log(y)$

3. Evaluate

   (a) $\sum_{k=0}^{10} \frac{1}{4^k}$ These are the first 11 terms of a geometric series with $r = 1/4$.

$$\sum_{k=0}^{10} \frac{1}{4^k} = \frac{1 - (1/4)^{11}}{1 - 1/4} \approx 1.333$$

.

   (b) $\frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \frac{1}{24} + \frac{1}{48} + \cdots$ This is 1/3 times a geometric series with ratio $r = 1/2$:

$$\frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \frac{1}{24} + \frac{1}{48} + \cdots = \frac{1}{3}\sum_{k=0}^{\infty} \frac{1}{2^k} = \frac{1}{3}\frac{1}{1 - 1/2} = \frac{2}{3}$$

4. Use integration by parts to evaluate $\int_0^1 x\exp(x)dx$.
Integration by parts: $\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx$.

$$\int_0^1 x\exp(x)dx = x\exp(x)\Big|_0^1 - \int_0^1 \exp(x)dx$$
$$= \exp(1) - 0 - [\exp(x)]_0^1 = \exp(1) - (\exp(1) - 1) = 1$$

5. Differentiate

(a) $\exp(-x^2)$ Using chain rule:

$$\frac{d}{dx}\exp(-x^2) = -2x\exp(-x^2)$$

(b) $\log(x)$

$$\frac{d}{dx}\log(x) = \frac{1}{x}$$

(c) $\log(x^3)$ Using chain rule:

$$\frac{d}{dx}\log(x^3) = \frac{3x^2}{x^3} = \frac{3}{x}$$

Or simply using the fact that $\log(x^3) = 3\log(x)$

(d) $x^3\exp(-x)$ Using product rule:

$$\frac{d}{dx}x^3\exp(-x) = 3x^2\exp(-x) - x^3\exp(-x)$$

(e) $\frac{x}{\exp(x)}$
Quotient rule: $\frac{d}{dx}\frac{u(x)}{v(x)} = \frac{u'(x)v(x)-u(x)v'(x)}{(v(x))^2}$ wherever $v(x)$ is nonzero.

$$\frac{d}{dx}\frac{x}{\exp(x)} = \frac{\exp(x) - x\exp(x)}{(\exp(x))^2} = \frac{1-x}{\exp(x)}$$

6. Use integration by substitution so show that $\int_0^1 \frac{1}{\sqrt{z}(1+\sqrt{z})}dz = 2\log(2)$
Substitute: $u = \sqrt{z}$. Then

$$\frac{du}{dz} = \frac{1}{2\sqrt{z}} \Rightarrow dz = 2\sqrt{z}\,du = 2u\,du$$

Also, when $0 < z < 1$ then $0 < u < 1$ so

$$\int_0^1 \frac{1}{\sqrt{z}(1+\sqrt{z})}dz = \int_0^1 \frac{2u}{u(1+u)}du = \int_0^1 \frac{2}{1+u}du$$

You may recognize this as $2\log(1+u)$ or you can do another substitution: Set $v = 1+u$, then

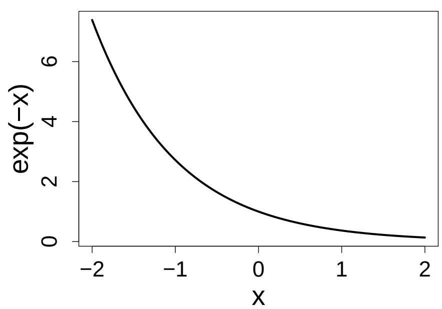
$$\frac{dv}{du} = 1 \Rightarrow du = dv$$

When $0 < u < 1$ then $1 < v < 2$ so

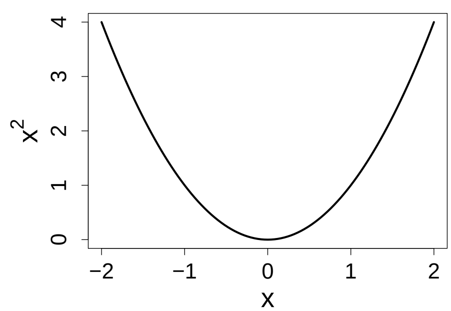
$$\int_0^1 \frac{2}{1+u} du = \int_1^2 \frac{2}{v} dv = \Big[2\log(v)\Big]_1^2$$
$$= 2\log(2) - 2\log(1) = 2\log(2)$$

7. Sketch rough plots of the following functions, indicating at least the point of intersection with the y axis

   (a) $f(x) = \exp(-x)$, $-\infty < x < \infty$



   (b) $f(x) = x^2$, $-\infty < x < \infty$



   (c) $f(x) = \exp(-x^2)$, $-\infty < x < \infty$