Robust Bayesian Regression

Readings: Hoff Chapter 9, West JRSSB 1984, Fúquene, Pérez & Pericchi 2015

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Using BAS

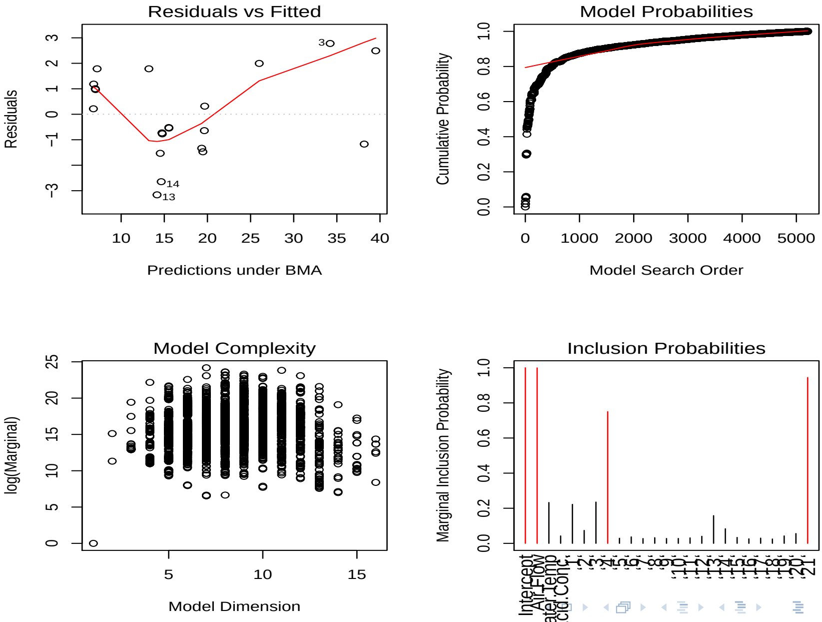
```
library(MASS)
data(stackloss)
n = nrow(stackloss)
stack.out = cbind(stackloss, diag(n))

library(BAS)
BAS.stack = bas.lm(stack.loss ~ ., data=stack.out,
                   prior="hyper-g-n", a=3,
                   modelprior=tr.beta.binomial(1, 1,15) ,
                   method="MCMC", MCMC.it=200000)
```

Output

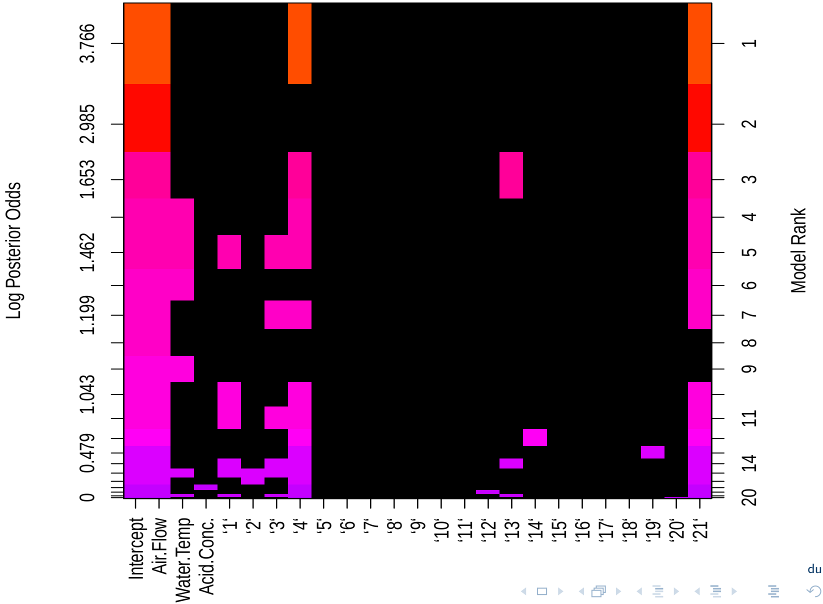
	P(B != 0 | Y)	model 1	model 2	model 3	model 4	model 5
Intercept	1.00	1.00	1.00	1.00	1.00	1.00
Air.Flow	1.00	1.00	1.00	1.00	1.00	1.00
Water.Temp	0.23	0.00	0.00	0.00	1.00	1.00
Acid.Conc.	0.04	0.00	0.00	0.00	0.00	0.00
‘1‘	0.22	0.00	0.00	0.00	0.00	1.00
‘2‘	0.07	0.00	0.00	0.00	0.00	0.00
‘3‘	0.24	0.00	0.00	0.00	0.00	1.00
‘4‘	0.75	1.00	0.00	1.00	1.00	1.00
‘5‘	0.03	0.00	0.00	0.00	0.00	0.00
‘6‘	0.04	0.00	0.00	0.00	0.00	0.00
‘7‘	0.03	0.00	0.00	0.00	0.00	0.00
‘8‘	0.03	0.00	0.00	0.00	0.00	0.00
‘9‘	0.03	0.00	0.00	0.00	0.00	0.00
‘10‘	0.03	0.00	0.00	0.00	0.00	0.00
‘11‘	0.03	0.00	0.00	0.00	0.00	0.00
‘12‘	0.04	0.00	0.00	0.00	0.00	0.00
‘13‘	0.16	0.00	0.00	1.00	0.00	0.00
‘14‘	0.08	0.00	0.00	0.00	0.00	0.00
‘15‘	0.03	0.00	0.00	0.00	0.00	0.00
‘16‘	0.03	0.00	0.00	0.00	0.00	0.00
‘17‘	0.03	0.00	0.00	0.00	0.00	0.00
‘18‘	0.02	0.00	0.00	0.00	0.00	0.00
‘19‘	0.04	0.00	0.00	0.00	0.00	0.00
‘20‘	0.06	0.00	0.00	0.00	0.00	0.00
‘21‘	0.94	1.00	1.00	1.00	1.00	1.00
BF		0.13	0.01	0.08	0.07	1.00
PostProbs		0.24	0.11	0.03	0.02	0.02
R2		0.96	0.93	0.97	0.97	0.99
dim		4.00	3.00	5.00	5.00	7.00
logmarg		22.17	19.43	21.68	21.57	24.18

BAS


Residuals vs Fitted
Residuals
Predictions under BMA
3
14
13
Model Probabilities
Cumulative Probability
Model Search Order
Model Complexity
log(Marginal)
Model Dimension
Inclusion Probabilities
Marginal Inclusion Probability
Intercept
Air.Flow

duke.eps

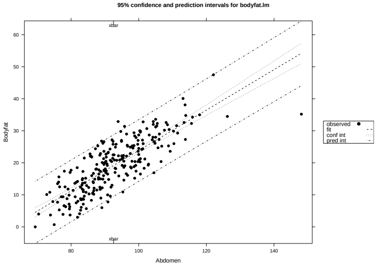
BAS

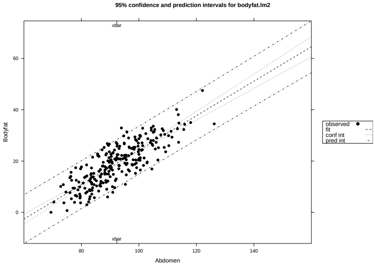


Log Posterior Odds
3.766
2.985
1.653
1.462
1.199
1.043
0.479
0
Model Rank
1
2
3
4
5
6
7
8
9
11
14
20
Intercept
Air.Flow
Water.Temp
Acid.Conc.
'1'
'2'
'3'
'4'
'5'
'6'
'7'
'8'
'9'
'10'
'11'
'12'
'13'
'14'
'15'
'16'
'17'
'18'
'19'
'20'
'21'

Body Fat Data: Intervals w/ All Data

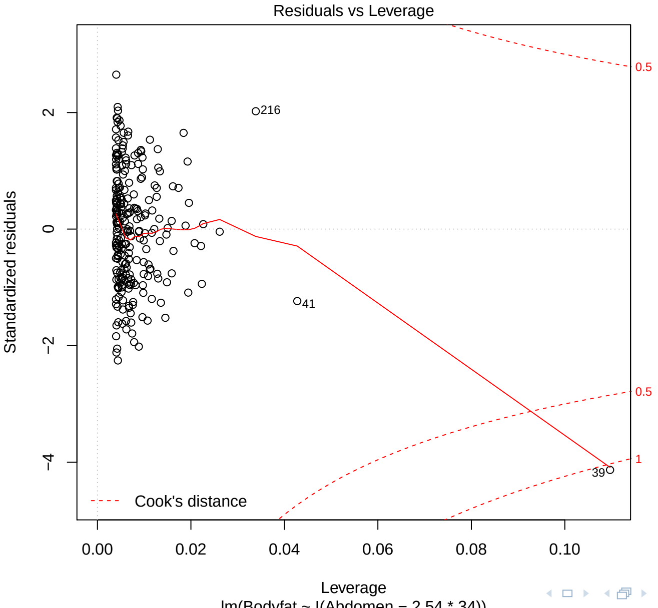
Response % Body Fat and Predictor Waist Circumference





Which analysis do we use? with Case 39 or not – or something different?

Cook's Distance



duke.eps

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- Model Averaging to Account for Model Uncertainty?
- Full model $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{I}_n \delta + \epsilon$
- 2^n submodels $\gamma_i = 0 \Leftrightarrow \delta_i = 0$
- If $\gamma_i = 1$ then case i has a different mean "mean shift" outliers.

Mean Shift = Variance Inflation

- Model $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{I}_n\delta + \epsilon$
- Prior
  $\delta_i \mid \gamma_i \sim N(0, V\sigma^2\gamma_i)$
  $\gamma_i \sim \text{Ber}(\pi)$

Then ϵ_i given σ^2 is independent of δ_i and

$$\epsilon_i^* \equiv \epsilon_i + \delta_i \mid \sigma^2 \begin{cases} N(0, \sigma^2) & wp \quad (1-\pi) \\ N(0, \sigma^2(1+V)) & wp \quad \pi \end{cases}$$

Model $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \epsilon^*$ "variance inflation"
$V + 1 = K = 7$ in the paper by Hoeting et al. package `BMA`

Simultaneous Outlier and Variable Selection

```
MC3.REG(all.y = bodyfat$Bodyfat, all.x = as.matrix(bodyfat$Abdom
        num.its = 10000, outliers = TRUE)

Model parameters: PI=0.02 K=7 nu=2.58 lambda=0.28 phi=2.85

  15  models were selected
 Best  5  models (cumulative posterior probability =  0.9939):

           prob     model 1  model 2  model 3  model 4  model 5
variables
  all.x    1          x        x        x        x        x
outliers
  39       0.94932  x        x        .        x        .
  204      0.04117  .        .        .        x        .
  207      0.10427  .        x        .        .        x

post prob           0.815    0.095    0.044    0.035    0.004
```

Change Error Assumptions

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$$p(\alpha, \beta, \phi \mid Y) \propto \phi^{n/2-1} \prod_{i=1}^{n} \left(1 + \frac{\phi(y_i - \alpha - \beta x_i)^2}{\nu}\right)^{-\frac{(\nu+1)}{2}}$$

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Bounded Influence - West 1984 (and references within)

Treat σ^2 as given, then *influence* of individual observations on the posterior distribution of $\boldsymbol{\beta}$ in the model where $\mathsf{E}[\mathbf{Y}_i] = \mathbf{x}_i^T \boldsymbol{\beta}$ is investigated through the score function:

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An outlying observation y_j is accommodated if the posterior distribution for $p(\boldsymbol{\beta} \mid \mathbf{Y}_{(i)})$ converges to $p(\boldsymbol{\beta} \mid \mathbf{Y})$ for all $\boldsymbol{\beta}$ as $|\mathbf{Y}_i| \to \infty$. Requires error models with influence functions that go to zero such as the Student t (O'Hagan, 1979)

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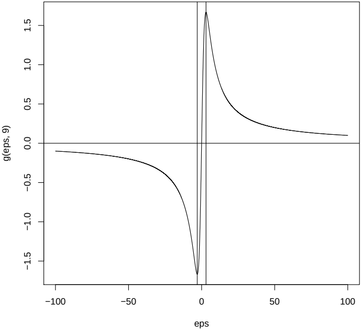

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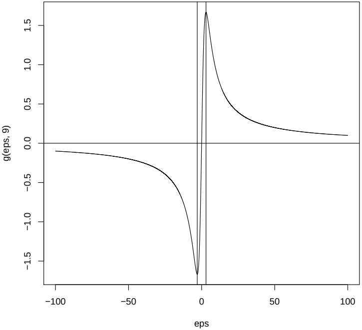

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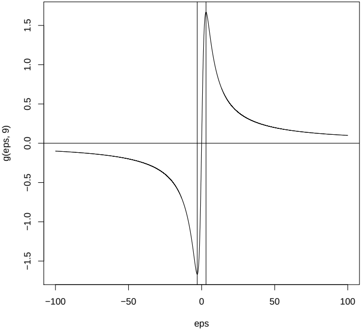

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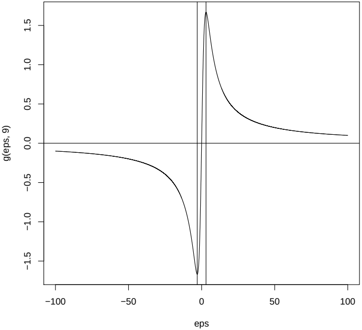

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Integrate out "latent" λ's to obtain marginal distribution.

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p((\alpha, \beta, \phi, \lambda_1, \ldots, \lambda_n \mid Y) \propto \qquad \phi^{n/2} \exp\left\{-\frac{\phi}{2} \sum \lambda_i (y_i - \alpha - \beta x_i)^2\right\} \times
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JAGS

Just Another Gibbs Sampler (and more)

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May do this through ordinary text files or use the functions in `R2jags` to specify model, data, and initial values then call `jags`.

Model Specification via R2jags

```
rr.model = function() {
  for (i in 1:n) {
    mu[i] <- alpha0 + alpha1*(X[i] - Xbar)
    lambda[i] ~ dgamma(9/2, 9/2)
    prec[i] <- phi*lambda[i]
    Y[i] ~ dnorm(mu[i], prec[i])
  }
  phi ~ dgamma(1.0E-6, 1.0E-6)
  alpha0 ~ dnorm(0, 1.0E-6)
  alpha1 ~ dnorm(0,1.0E-6)
}
```

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- Normal distributions are parameterized using *precisions*, so `dnorm(0, 1.0E-6)` is a $N(0, 1.0 \times 10^6)$
- uses `for` loop structure as in R for model description but coded in C++ so is fast!

Data

A list or rectangular data structure for all data and summaries of data used in the model

```
bf.data = list(Y = bodyfat$Bodyfat,
               X=bodyfat$Abdomen)
bf.data$n = length(bf.data$Y)
bf.data$Xbar = mean(bf.data$X)
```

Specifying which Parameters to Save

The parameters to be monitored and returned to R are specified with the variable `parameters`

```
parameters = c("beta0", "beta1", "sigma",
               "mu34", "y34", "lambda[39]")
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- To save a whole vector (for example all lambdas, just give the vector name)

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- All of the above (except lambda) are calculated from the other parameters. (See R-code for definitions of these parameters.)
- `lambda[39]` saves only the 39th case of λ
- To save a whole vector (for example all lambdas, just give the vector name)

Running jags from R

```
bf.sim = jags(bf.data, inits=NULL, par=parameters,
              model=rr.model,
              n.chains=2, n.iter=10000,
              )
```

Output

	mean	sd	2.5%	50%	97.5%
beta0	-41.70	2.75	-46.91	-41.67	-36.40
beta1	0.66	0.03	0.60	0.66	0.71
sigma	4.48	0.23	4.05	4.46	4.96
mu34	15.10	0.35	14.43	15.10	15.82
y34	14.94	5.15	4.37	15.21	24.65
lambda[39]	0.33	0.16	0.11	0.30	0.72

95% HPD interval for expected bodyfat $(14.5, 15.8)$

95% HPD interval for bodyfat $(5.1, 25.3)$

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Case 39 down weighted by λ_{39}

Full Conditional for λ_j

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&\qquad \prod_{i=1}^{n} \lambda_i^{\frac{\nu+1}{2}-1} \exp(-\lambda_i \frac{\nu}{2})
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$$
\lambda_j \mid \text{rest}, Y \sim G\left(\frac{\nu+1}{2}, \frac{\phi(y_j - \alpha - \beta x_j)^2 + \nu}{2}\right)
$$

Weights

Under prior $E[\lambda_i] = 1$

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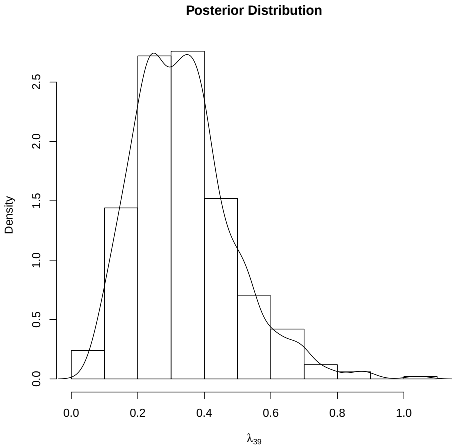
Under posterior, large residuals are down-weighted (approximately those bigger than $\sqrt{\nu}$)



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