Statistics 360/601 – Modern Bayesian Theory

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Lecture 5 - Sept 11, 2018

One parameter families



How often do we see Poisson data?

Google	poisson model count data
Scholar	About 400,000 results (0.12 sec)
Articles	Zero-truncated and zero-inflated models for count data AF Zuur, EN leno, NJ Walker, <u>AA Saveliev</u> Mixed effects models, 2009 - Springer However, the ZIP model uses a Poisson distribution for the counts, and the ordinary P
Case law	GLM applied on If the overdispersion in a Poisson GLM is caused by the excessive nur
My library	zeros, then the is not caused by the zeros, then the ZIP is not the appropriate model eith Cited by 7587 Related articles All 14 versions Cite Save More
Any time	[HTML] Differential expression analysis for sequence count data
Since 2017	<u>S Anders, w Huber</u> - Genome biology, 2010 - genomebiology.biomedcentral.com Thus, the count value K ii. conditioned on R ii = r ii. is Poisson distributed with rate s i
Since 2016	in Equation (3). Furthermore, if the higher moments of R ij are modeled according to a gam
Since 2013	distribution, the marginal distribution of K ij is The model has three sets of parameter
Custom range	Cited by 5983 Related articles All 45 versions Web of Science: 3955 Cite Save Me
Sort by relevance	Econometric models based on count data. Comparisons and application
Sort by date	SOME ESTIMATORS AND TESTS AC Cameron, PK Trivedi - Journal of applied econometrics, 1986 - Wiley Online Library
	Var(y, $ X_{1}, 8\rangle = p1 + up!$, for given I where the distribution for y, under HA may not nece
include patents	the variance of v. equals pl when v. is Poisson distributed, tests of H a against HA are
include citations	Cited by 1903 Related articles All 8 versions Cite Save More

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- Observed data: 3 deaths in the city.
- Crude estimate: 1.5 deaths per 100,000.

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- y = 3, x = 2 and θ is unknown.

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Comparison of Gamma with mean 0.6

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 (What did we do? Set the prior mean α/β = 0.6 and play with the parameters)

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- ▶ In this data example the posterior is *Gamma*(6,7)
- Posterior mean is 0.86, shrinking the data substantially towards the global mean.