STA 214 Assignment 4 Spring 1997

Readings: ...

Review the latest classes on multivariate distributions, together with the handouts on the multivariate normal and Gelman (sect 3.6) on normal models. Be ready for discussion of multivariate normal mixtures as in the paper by M Lavine & M West and the other short article by Christian Robert, in the coursepack.

The course homepage has a new item on mixture priors, a short (one page) outline of basic theory that underlies important approaches to testing, model assessment, etc. Check it out – it helps with Exercise 2 below. We'll discuss related issues in class later in the semester.

Please hand in solutions to the following at the start of class on Thursday 13th February **(but get started now!)**

Exercise 1

Pairs of observations on intensity and duration of a neural signal are made repeatedly on experimental tissue. The sample mean vector and covariance matrix of $n = 50$ observations are

$$\bar{y} = \begin{pmatrix} 2.145 \\ 7.590 \end{pmatrix} \quad \text{and} \quad \hat{\Sigma} = \begin{pmatrix} 0.450 & 0.275 \\ 0.275 & 0.981 \end{pmatrix}.$$

Assume a multivariate normal random sampling model, and Jeffreys' (reference) prior on the mean and variance matrix μ and Σ. Write S-Plus code to sample the joint posterior for (μ, Σ). Add a line of code to sample the posterior *predictive* distribution for a further observation y_{n+1}.

Summarise the posterior for μ and the predictive for y_{n+1} through plots of marginal (histograms) and joint (2-d histograms, scatter plots, or contours, etc) of the posterior/predictive samples, respectively.

n.b., (Check the course homepage to pick up my S-Plus function for sampling the inverse Wishart distribution to help with this.)

Exercise 2

A standard mixture prior problem – the "classical" null hypothesis testing problem.

Independent (univariate) normal observations of known variance v but unknown mean μ are recorded; a sample of size n is made, so that the sample mean $\bar{y}$ is sufficient for μ and the likelihood function for μ is $A\exp(-(\mu - \bar{y})^2/(2v/n))$ with $A = 1/\sqrt{2\pi v/n}$. The "null hypothesis" is $\mu = 0$, with "alternative" $\mu \neq 0$. An investigator takes the mixture prior

$$p(\mu) = (1 - \pi)I_0(\mu) + \pi N(\mu|0, w).$$

This says that $Pr(\mu = 0) = 1 - \pi$ and the normal component of the prior is that for $(\mu|\mu \neq 0)$. Note that for large w this says that "anything goes" under the alternative hypothesis.

(a) Calculate the Bayes' factor (ratio of posterior to prior odds) on $\mu = 0$ versus $\mu \neq 0$ (i.e., the Bayes' factor in favour of the null hypothesis) given the datum $\bar{y}$, and show that it can be written as

$$(1 + nw/v)^{1/2} \exp(-z^2/(2a))$$

where $a = 1 + v/(nw)$ and z is the standard $Z-$test statistic $z = \bar{y}\sqrt{n/v}$.

(N.B., *A useful fact:*

$$\int A \exp(-n(\mu - \bar{y})^2/2v) N(\mu|0, w) d\mu = \exp(-\bar{y}^2/(2(w + v/n)))/\sqrt{(2\pi(w + v/n))}.$$

You should be able to prove this without doing the integration–use it for this question in any case).

(b) Suppose $n = 50$, $v = 1$ and that we observe $z = 1.96$. Graph the Bayes' factor as a function of $w^{1/2}$ over $1 < w^{1/2} < 4$. Graph the corresponding posterior probability on $\mu = 0$ when $\pi = 0.5$. Recall that standard testing would reject the null hypothesis at significance level 0.05 with this observed z value. Would this investigator be likely to agree that this is a sensible conclusion, and how do her views depend on the value she specifies for w?
