Assignment 6

1. Consider the following nonconjugate, nonlinear regression model. The data are length(x_i) and age (y_i) for 27 captured dugongs (sea cows).

$$\begin{aligned} y_i &\sim N(y_i|\mu_i, v), \quad i = 1, \dots, n = 27 \\ \mu_i &= \alpha - \beta\gamma^{x_i}, \\ & \alpha, \beta > 1; 0 < \gamma < 1, \end{aligned}$$

with a standard noninformative prior $p(\alpha, \beta, v, \gamma)$.

1a. Draw a graphical representation of the probability model, using nodes for

$$\alpha, \beta, v, \gamma, \mu_i, x_i, y_i.$$

1b. Augment the model by adding an error-in-variables component for x_i, i.e., introduce a "proxy" w_i with

$$w_i \sim N(w_i|x_i, \rho).$$

Instead of observing x_i we only observe w_i, maybe because it is difficult to measure the exact length of a dugong, or it is only an estimate from eye judgement. Modify your graphical probability model from 1a accordingly (just add with color to the same figure if you wish).

1c (Optional!): If you have some extra time over spring break, get familiar with BUGS. Try to implement a Gibbs sampler in this problem using BUGS. BUGS is a S-plus look alike language to define Bayesian models, and implement posterior simulation. Borrow the BUGS manual from me (or any student taking STA 216) if you want to copy it. We will (most probably) use BUGS for some of the models which we will discuss later. We will still talk about BUGS in class. But the best way to learn it is really "hands-on".

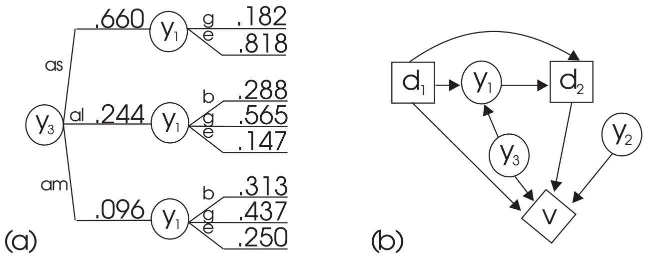
The data for this example is in `/usr/local/bugs05/examples/dugongs/dugongs.dat`. You can find BUGS code in `dugongs.bug` in the same directory (using the input file `dugong.in`). See/email me if you don't have access to the ISDS system.

Note: Bugs uses different notation for the normal distribution. Instead of mean and variance they use mean and *precision*, i.e. replace $N(y_i|\mu_i, v)$ by $N(y_i|\mu_i, \tau)$ where $\tau = 1/v$.

2. The reactor problem (Covaliu and Oliver 1995; Bielza and Shenoy 1996).

An electric company has to decide (d_2) whether to buy a conventional (c) or an advanced (a) reactor. The advanced reactor will lead to bigger benefits if successful, but entails bigger risks. The benefits in monetary units for c are 8, if successful (cs) (with probability .98), and -4, if not (0.02) (cf). The benefits for the advanced one are 12, if successful (as) (.66), -6, if a minor accident happens (al) (.244), and -10, if a major accident happens (am) (.096). The company may ($d_1 = 1$) or may not ($d_1 = 0$) commission a test of the advanced reactor. This involves cost 1. The result of the test may be bad (b), good (g) or excellent (e), with probabilities given in the causal probabilistic model in Figure 1a. If the test results are bad, the Nuclear Regulatory Commission will not permit an advanced reactor, and a conventional one will be constructed. Figure 1b provides an ID for the reactor problem.

The problem consists of two decision nodes $d = (d_1, d_2)$, where $d_1 \in \{0, 1\}$ and $d_2 \in \{c, a\}$. There are three data nodes: $y_1 \in \{b, g, e\}$, $y_2 \in \{cs, cf\}$ and $y_3 \in \{as, al, am\}$, corresponding to the outcome of the test, accidents at a conventional reactor and accidents at an advanced reactor type, respectively. There are no unknown parameters θ. The decision d_2 can actually depend on



(a) (b)

Figure 1: The reactor problem. a) Probabilistic model. b) Initial Influence Diagram.

the outcome of y_1 and the earlier decision d_1, i.e. the problem is sequential. To reduce it to a non-sequential problem we replace the second decision by a list of decision nodes with a new separate node corresponding to each possible scenario of the predecessor nodes.

Hence, we replace d_2 by the pair (d_2^{gn}, d_2^e), where d_2^e is the reactor type to be chosen if $d_1 = 1, y_1 = e$, and d_2^{gn} is the type to be chosen otherwise. Note that the decision under $(d_1 = 1, y_1 = b)$ is fixed by the regulatory agency to $d_2 = c$. The decision node d_2^{gn} could be further split into d_2^g, d_2^n corresponding to the decisions to be taken if $(d_1 = 1, y_1 = g)$ and $(d_1 = 0)$, respectively. However, this is not necessary, since d_1 already separates these two scenarios in the following sense: $(d_1 = 1, d_2^{gn})$ is the decision if a test is commissioned and a "good" outcome is recorded; $(d_1 = 0, d_2^{gn})$ is the decision if no test is commissioned. The ID in Figure 2 summarizes the problem.

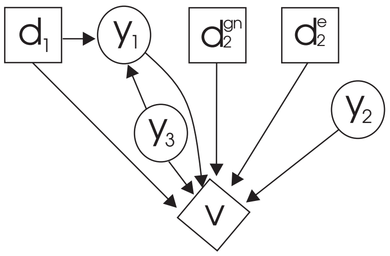


Figure 2: Modified ID for the reactor problem.

2a. Use "averaging out" (= marginalization), "arc reversal" (= Bayes' theorem) and maximization to solve the influence diagram given in Figure 2.