FINAL EXAM (ver 1)

STA 110A
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Name: ______________________________

1. This is an open book and open notes exam.
2. You show work and explain the answer in order to receive credit.
3. The exam carries 100 points.
4. The points assigned to each problem are indicated at the end of that problem. Use them to plan your time. You have 3 hours to finish.
5. The exam has 6 + 1 problems. Make sure your copy has all of them.

Problem	1	2	3	4	5	6	(7)	Total
Score	/15	/20	/20	/15	/15	/15		/100

Duke Wear Pricing Practices.[1] Ever since the Duke Blue Devils won back-to-back National Basketball championship, the demand for Duke sweatshirts has skyrocketed not only at Duke, but across the nation as well. However after three years of buying their sweatshirts on campus, many students have found that their friends at other schools often purchase twice as many Duke shirts from department stores far from Duke. This has led many students to complain that they are being unfairly overcharged because Duke sweatshirts are apparently priced higher on campus than they are off campus and elsewhere in the United States.

One particularly disgruntled group of students in their STA 110 project wanted to test the hypothesis that higher retail prices are being charged for sweatshirts in Duke stores than are charged off campus. They obtain random samples of 72 retail sweatshirt sales on campus and 55 such retail sales from stores off campus over the same time period and for the same style of sweatshirts. The following data were obtained:

Duke Sales	Off-Campus Sales
$\bar{X}_1 = \$49.35$	$\bar{X}_2 = \$43.05$
$s_1 = \$6.70$	$s_2 = \$7.98$

(a) Do these data provide sufficient evidence to support the students' claim that the mean sales price of Duke sweatshirts is higher at Duke than it is off campus? State the null and the alternative hypothesis and perform the test at $\alpha = 0.05$.

(b) Since samples are large, you can use z approximation for the exact t test in (a). Calculate approximate p-value for the test in (a).

[1] From STA110 student projects

Clover Varieties. Six plots each of five varieties of Clover were planted at the Danbury Experiment Station in North Carolina. Yields in tons per acre were as follows:

Variety	*Yield*
Spanish	2.79, 2.26, 3.09, 3.01, 2.56, 2.82
Evergreen	1.93, 2.07, 2.45, 2.20, 1.86, 2.44
Commercial Yellow	2.76, 2.34, 1.87, 2.55, 2.80, 2.21
Madrid	2.31, 2.30, 2.49, 2.26, 2.69, 2.17
Wisconsin A46	2.39, 2.05, 2.68, 2.96, 3.04, 2.60

The following MINITAB output is obtained:

```
MTB > oneway c1 c2;
SUBC>   fisher.

ANALYSIS OF VARIANCE ON C1
SOURCE     DF        SS        MS        F        p
C2          4    1.2784    0.3196     3.53    0.020
ERROR      25    2.2619    0.0905
TOTAL      29    3.5403
                                   INDIVIDUAL 95 PCT CI'S FOR MEAN
                                   BASED ON POOLED STDEV
 LEVEL      N      MEAN     STDEV  -------+---------+---------+---------
     1      6    2.7550    0.3052                      (--------*-------)
     2      6    2.1583    0.2510   (-------*-------)
     3      6    2.4217    0.3549           (--------*-------)
     4      6    2.3700    0.1884          (-------*-------)
     5      6    2.6200    0.3671                  (-------*--------)
                                   -------+---------+---------+---------
POOLED STDEV =   0.3008                 2.10      2.40      2.70

FISHER'S multiple comparison procedure

        Nominal level = 0.0500
    Family error rate = 0.268
Individual error rate = 0.0500

Critical value = 2.060

Intervals for (mean of column group) - (mean of row group)

             1        2        3        4

    2   0.2389
        0.9544
```

```
3  -0.0244  -0.6211
    0.6911   0.0944

4   0.0273  -0.5694  -0.3061
    0.7427   0.1461   0.4094

5  -0.2227  -0.8194  -0.5561  -0.6077
    0.4927  -0.1039   0.1594   0.1077
```

(i) What is ANOVA? What are assumptions for ANOVA?

(ii) Test the hypothesis that the mean yields for the five clover varieties are the same. Take $\alpha = 5\%$. What happens if your α is 1%.

(iii) Which means are different at 5% level? Why?

(iv) Is the hypothesis $H_0 : 3(\mu_1 + \mu_5) = 2(\mu_2 + \mu_3 + \mu_4)$ a contrast? Why? If yes, test it against the two sided alternative, at $\alpha = 5\%$ level.

Comet Bennett. Bacos[2] reported observations made on comet Bennett, which are given in the table below.

Heliocentric Dist. (R)	0.685	0.720	0.735	0.750	0.798	0.810	0.828	0.950
Reduced Vis. Mag. (H)	1.72	1.80	2.53	2.15	2.81	2.92	2.80	3.82
Heliocentric Dist. (R)	0.988	1.210	1.228	1.224	1.267	1.295	1.312	1.330
Reduced Vis. Mag. (H)	3.77	5.06	5.21	5.19	5.39	5.61	5.57	5.89

(i) The coefficient of linear correlation, r, between reduced visual magnitude (H) and the heliocentric distance (R) is found to be 0.993. Comment on appropriateness of a linear model. (One or two sentences)

The MINITAB output gives the regression equation $\hat{H} = \hat{\beta}_0 + \hat{\beta}_1 R$.

```
The regression equation is
H = - 2.17 + 6.01 R

Predictor       Coef       Stdev    t-ratio        p
Constant     -2.1734      0.2004     -10.85    0.000
R             6.0145      0.1933      31.11    0.000

s = 0.1864      R-sq = 98.6%     R-sq(adj) = 98.5%

SOURCE        DF          SS          MS         F         p
Regression     1      33.615      33.615    967.97     0.000
Error         14       0.486       0.035
Total         15      34.101
```

(i) Test the hypothesis $H_0 : \beta_1 = 5.5.$ against the alternative $H_1 : \beta_1 > 5.5$ at the level of significance $\alpha = 0.05$.

(ii) Find 98%-Confidence interval for the intercept β_0.

(iii) The "fit" for $R = 1.1$ is given in the following part of MINITAB output:

```
    Fit  Stdev.Fit         95% C.I.            95% P.I.
 4.4426     0.0499   ( 4.3356, 4.5495)  ( 4.0287, 4.8564)
```

Find a 97% confidence interval for the mean fit. Note that the 95% confidence interval is given: $(4.3356, 4.5495)$. Also, all information needed for solving (iii) is given in the output (except cut-points).

[2]Bacos, G. (1973). Photoelectric observations of comet Bennett. *J. Roy. Astron. Soc. Can., 67, 183-189.*

Butterflies. The following data were extracted from a larger study by Brower[3] on a speciation in a group of swallowtail butterflies. Morphological measurements are in millimeters coded $\times$ 8. (Y_1 - length of 8th tergile, Y_2 - length of superuncus)

Species	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2
Papilio	24	14	21	15	20	17.5	21.5	16.5
multicaudatus	21.5	16	25.5	16	25.5	17.5	28.5	16.5
	23.5	15	22	15.5	22.5	17.5	20.5	19
	21	13.5	19.5	19	26	18	23	17
	21	18	21	17	20.5	16	22.5	15.5
Papilio	20	11.5	21.5	11	18.5	10	20	11
rutulus	19	11	20.5	11	19.5	11	19	10.5
	21.5	11	20	11.5	21.5	10	20.5	12
	20	10.5	21.5	12.5	17.5	12	21	12.5
	21	11.5	21	12	19	10.5	19	11
	18	11.5	21.5	10.5	23	11	22.5	11.5
	19	13	22.5	14	21	12.5	19.5	12.5

The correlation coefficients separately for each species are $r_1 = -0.11$(for *P. multicaudatus*) and $r_2 = 0.176$(for *P. rutulus*)

Test significance of each. Test whether the two correlation coefficients differ significantly.

HINT: The following example may be useful: *Test for the difference between two correlation coefficients.* The test statistic for $H_0 : \rho_1 = \rho_2$ is

$$z = \frac{w_1 - w_2}{\sqrt{\frac{1}{n_1-3} + \frac{1}{n_2-3}}}$$

where $w_i = \frac{1}{2}\ln\frac{1+r_i}{1-r_i}$, $i = 1, 2$ and n_1, n_2 number of pairs in the first, second sample. (You may use the table on page 478 (Iman, Text)) A good approximation for the test cut-points are quantiles of normal distribution.

• The correlation between body weight and wing length in *Drosophila pseudoobscura* was found [4] to be 0.52 in a sample of $n_1 = 39$ at the Grand Canyon and 0.67 in a sample of $n_2 = 20$ at Flagstaff, Arizona.

Grand Canyon: $w_1 = 0.5763$ Flagstaff: $w_2 = 0.8107$

The test statistic for $H_0 : \rho_1 = \rho_2$ is: $z = \frac{0.5763-0.8107}{\sqrt{1/36+1/17}} = -0.7965$.

p-value against two sided hypothesis is $2\Phi(-0.7965) = 0.4257$. Conclusion: Do not reject null hypothesis.

[3] Brower, L. P. (1959). Speciation in butterflies of the *Papilio glaucus* group. I Morphological Relationships and hybridizations. *Evolution 13, 40-63.*

[4] Sokoloff, A. (1966). Morphological variation in natural and experimental populations of *Drosophila pseudoobscura* and *Drosophila persimilis. Evolution, 20, 49-71.*

Bayes and Bats. By careful examination of sound and film records it is possible to measure the distance at which a bat first detects an insect. The measurements are modeled by normal distribution $N(\theta, 10^2)$, where θ is the unknown mean distance (in cm).

Experts believe that the prior suitably expressing uncertainty about θ is $\theta \sim N(50, 10^2)$.

Three measurements are obtained: 62, 52, and 68.

(a) Find the posterior distribution of θ given the observations.

(b) Test the hypothesis H_0 that $\theta \geq 50$ in Bayesian fashion.

(c) What is the 95% credible set for θ.

Taste of Cheese. As cheddar cheese matures a variety of chemical processes take place. The taste of mature cheese is related to the concentration of several chemicals in the final product. In a study of cheddar cheese from LaTrobe Valley of Victoria, Australia, samples of cheese were analyzed for their chemical composition and where subjected to taste tests. The table below presents data[5] for one type of cheese manufacturing process. *Taste* is the response variable of interest. The taste scores where obtained by combining scores from several tasters. Three of the chemicals whose concentrations were measured were *acetic acid, hydrogen sulfide,* and *lactic acid.* For acetic acid and hydrogen sulfide log transformations were taken.

Taste	Acetic	H2S	Lactic	Taste	Acetic	H2S	Lactic
12.3	4.54	3.13	0.86	20.9	5.16	5.04	1.53
39.0	5.37	5.44	1.57	47.9	5.76	7.59	1.81
5.6	4.66	3.81	0.99	25.9	5.70	7.60	1.09
37.3	5.89	8.73	1.29	21.9	6.08	7.97	1.78
18.1	4.90	3.85	1.29	21.0	5.24	4.17	1.58
34.9	5.74	6.14	1.68	57.2	6.45	7.91	1.90
0.7	4.48	3.00	1.06	25.9	5.24	4.94	1.30
54.9	6.15	6.75	1.52	40.9	6.37	9.59	1.74
15.9	4.79	3.91	1.16	6.4	5.41	4.70	1.49
18.0	5.25	6.17	1.63	38.9	5.44	9.06	1.99
14.0	4.56	4.95	1.15	15.2	5.30	5.22	1.33
32.0	5.46	9.24	1.44	56.7	5.86	10.20	2.01
16.8	5.37	3.66	1.31	11.6	6.04	3.22	1.46
26.5	6.46	6.92	1.72	.7	5.33	3.91	1.25
13.4	5.80	6.69	1.08	5.5	6.18	4.79	1.25

(1) Fill-in the ANOVA table:

```
The regression equation is
Taste = - 28.8 + _____ Acetic + 3.92 H2S + 19.6 Lactic

Predictor        Coef       Stdev    t-ratio        p
Constant        _____       _____      -1.46    0.155
Acetic          _____       4.450       0.07    0.942
H2S             3.924       1.245       3.15    0.004
Lactic         19.586       8.623       2.27    0.032

s = ____       R-sq = _______%     R-sq(adj) = 61.3%

Analysis of Variance
```

[5] Data from the experiments of G. T. Lloyd and E. H. Ramshaw, CISRO Food Research, Victoria, Australia. Published in Moore, D. and McCabe, G. *Introduction to the Practice of Statistics,* Freeman, 1989.

SOURCE	DF	SS	MS	F	p
Regression	____	_______	______	16.30	0.000
Error	____	2659.7	______		
Total	____	_______			

Fit	Stdev.Fit	95% C.I.	95% P.I.
22.13	3.42	(_______, ________)	(0.18, 44.08)

(2) Choose the "best" linear model in the problem. Explain your choice.

Best Subsets Regression of Taste

Vars	R-sq	Adj. R-sq	C-p	s	Acetic	H2S	Lactic
1	57.3	55.8	6.0	10.809		X	
1	49.6	47.8	11.8	11.745			X
2	65.3	62.7	2.0	9.9261		X	X
2	58.4	55.3	7.2	10.865	X	X	
3	65.3	61.3	4.0	10.114	X	X	X

Trial of the Pyx[6] (Optional Problem)

Since the early 13th century, coins struck by the Royal Mint in England have been evaluated for their metal content on a sample basis, in a ceremony called the Trial of the Pyx. This ceremony does not have much meaning anymore, but there was real money on the line back in the 1700s, because English coins were made of gold in those days. In 1799, for instance, the procedure went like this. One hundred gold coins called guineas were chosen at random from all of the coins made at the Mint that year, put in the Pyx (a ceremonial box), and weighed. The Master of the Mint, who was responsible for the quality of the coins, was allowed a margin of error, called the "remedy," which was set according to the manufacturing tolerances of the time.

In 1799 a guinea was supposed to weigh 128 grains (there are 360 grains in an ounce), so the 100 guineas in the Pyx should have weighed about 12800 grains. The remedy in those days was 1/400 of the expected amount, or 32 grains. If the actual weight of the coins in the Pyx differed from its expected value by more than the remedy on either the high or low side, the Master of the Mint was exposed to serious penalties. The British government had a vested interest in the coins not weighing too much, but the Master of the Mint had an incentive to make them weigh less than the standard, because he was allowed to keep the shortfall himself (as long as he was not caught by the Trial of the Pyx).

(i) If the Master of the Mint is honest and manufactures guineas that weigh exactly 128 grains on average, with a standard deviation of 1 grain, what is the chance that he will survive the Trial of the Pyx?

(ii) If instead he sets things up so that the guineas weigh only 127.7 grains on average (with the same standard deviation of 1 grain), what is the chance now that he will survive the Trial? If he does survive, how much gold can he expect to pocket in an average year in which he produces 100,000 guineas? Give or take how much? Show all your work.

[6] Author of the problem is Robert Gould, `rgould@stat.ucla.edu`