

### Quiz 3

1. A box contains 200 marbles. Marbles are made of glass (G), iron (I), or plastic (P), and they are colored blue (B), red (R), or yellow (Y). The content is described in the table given below:

|       | G  | I  | P  | Total |
|-------|----|----|----|-------|
| B     | 10 | 15 | 25 | 50    |
| R     | 35 | 50 | 15 | 100   |
| Y     | 15 | 10 | 25 | 50    |
| Total | 60 | 75 | 65 | 200   |

One marble is selected at random. What is the probability that the marble is

(a) red,

*Circle the number closest to your answer:*

[1 pt]      0.0      0.1      0.2      0.3      0.4      0.5      0.6      0.7      0.8      0.9      1.0

(b) yellow **and** made of iron,

*Circle the number closest to your answer:*

[1 pt]      0.0      0.1      0.2      0.3      0.4      0.5      0.6      0.7      0.8      0.9      1.0

(c) blue **or** made of plastic,

*Circle the number closest to your answer:*

[1 pt]      0.0      0.1      0.2      0.3      0.4      0.5      0.6      0.7      0.8      0.9      1.0

(d) blue, if it is known that it is made of iron,

*Circle the number closest to your answer:*

[2 pts]      0.0      0.1      0.2      0.3      0.4      0.5      0.6      0.7      0.8      0.9      1.0

(e) blue, if it is known that it is not made of glass,

*Circle the number closest to your answer:*

[2 pts]      0.0      0.1      0.2      0.3      0.4      0.5      0.6      0.7      0.8      0.9      1.0

(f) made of plastic, if it is known that it is yellow.

*Circle the number closest to your answer:*

[2 pts]      0.0      0.1      0.2      0.3      0.4      0.5      0.6      0.7      0.8      0.9      1.0

Your TA will go over the following problems:

- **Sherwood Story.** Robin Hood hits the target with the probability 0.8 while the Sheriff of Nottingham hits the target with the probability 0.6. They shoot 3 arrows each and all shots are independent of each other. (a) What event is more likely: **A**: Sheriff hits the target at least 2 times; or **B**: Robin hits the target at most 2 times. (b) They shoot in the following order: **S R S R S R**. What is the probability that the Sheriff scores before Robin?

- **Urns.** A probability-inclined executioner offers a convicted murderer a final chance to win his release. He gives the prisoner 8 chips (4 white, 4 black) and 2 indistinguishable urns<sup>1</sup>. He then instructs the prisoner to **place all 8 chips into the 2 urns, with the condition that each urn must contain at least one chip**. He also states that the prisoner can place the chips in any other manner the prisoner so chooses. He then explains that after placement of the chips, he will choose 1 urn at random, and from the chosen urn, randomly draw one chip. If the chip is white, the prisoner goes free. If chip is black ... the less said the better. (a) What is the sample space of the prisoners possible allocation scheme? (Hint: The order of the urns doesn't matter. That is, 4 white in urn 1 and 4 black in urn 2 is the same as 4 white in the urn 2 and 4 black in the urn 1. There are 12 possible allocations.) (b) Assuming the prisoner wants to live, how should he allocate the chips? What is the maximum probability of survival?

- **Circuit.** Suppose that each of the switches  $S_i$  in the following circuit is closed with probability  $p_i$  and open with probability  $q_i = 1 - p_i$ . Calculate the probability that a current will flow through the circuit, assuming that the switches act independently.

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ |
|-------|-------|-------|-------|-------|
| 0.4   |       | 0.6   | 0.8   |       |
| $q_1$ | $q_2$ | $q_3$ | $q_4$ | $q_5$ |
|       | 0.3   |       |       | 0.5   |

- **Simpson, but not O.J.** Some special cards are prepared for an unusual game. There are four varieties of cards: cards with red letter  $A$ , cards with blue letter  $A$ , cards with red letter  $B$  and cards with blue letter  $B$ . Deck **1** has forty of these cards, and the table below gives the content of this deck.

|     | Red | Blue |
|-----|-----|------|
| $A$ | 5   | 14   |
| $B$ | 5   | 16   |

Deck **2** also has forty of these cards, and the table below gives makeup of Deck **2**.

|     | Red | Blue |
|-----|-----|------|
| $A$ | 5   | 2    |
| $B$ | 20  | 13   |

Suppose the two decks can be combined to make a single deck of eighty cards.

1. Show:

$$P(A | \text{Red, Deck 1}) > P(A | \text{Blue, Deck 1})$$

$$P(A | \text{Red, Deck 2}) > P(A | \text{Blue, Deck 2})$$

$$P(A | \text{Red, Combined Deck}) < P(A | \text{Blue, Combined Deck})$$

2. Find:

$$P(\text{Red} | \text{Combined Deck, } A)$$

$$P(\text{Blue} | \text{Deck 1, } B)$$

3. Are the color and the letter on cards independent in Deck **2**?

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<sup>1</sup>There are 3 kinds of urns: Grecian urns, burial urns and probability urns.