STA213 **Fall 1999**

Midterm Exam

1. Suppose (X, Y) follows the bivariate normal distribution and $E(X) = E(Y) = 0$, $Var(X) = Var(Y) = 1$, $Cov(X, Y) = \rho$. Find $E(X^2|Y = y)$ and $Corr(X^2, Y)$.

2. Dave runs a small grocery store. The number of customers N who enter the store for a day follows $Poisson(40)$ and the amount of money spent by every customer who comes in the store follows $Exp(1/20)$. Find the Dave's expected revenue for a day and its variance.

3. After every lunch, Jaeyong chooses to do one of k activities such as chating with friends, preparing for STA213 lecture, chasing squirrels in the yard, etc... His utility for activity i is given by $x_i + \zeta_i$, where $x_1, \cdots, x_k$ are real numbers and $\zeta_1, \cdots, \zeta_k$ are iid with cdf

$$F(x) = \exp(-e^{-x}), x \in R.$$

In choosing one of k activities, he always maximize his utility, i.e. he chooses activity 1 if the utility of activity 1 is larger than all the other utilities. Show that the probability that he will choose activity 1 after lunch on the next Monday is given by

$$\frac{e^{x_1}}{\sum_{i=1}^{k} e^{x_i}}.$$

4. Suppose X, Y are random variables with $0 < Var(X^2) < \infty$ and $0 < Var(Y^2) < \infty$ and let $\rho = Corr(X, Y)$. Show that $|\rho| = 1$ if and only if $P(Y = aX + b) = 1$ for some $a, b \in R$.

5. A lottery company is considering the following game. Suppose N is the number of customers who play the game and P (dollars) is the amount of total prize. (N and P are fixed before starting the game.) The game is played by drawing six numbers without replacement from numbers $1, 2, \cdots, 10$. If a customer guesses all six numbers correctly (he/she does not need to guess the order of 6 numbers), he/she wins. If there are K customers who win the game, each customer will get P/K. (Of course, if $K = 0$, nobody gets the prize.) What is the expected winning prize for a customer?

6. Let X, Y be iid standard normal random variables. Find the joint density of (R, A), where R is the distance from the origin, $(0, 0)$, to (X, Y) and A is the angle between the x-axis and the line formed by connecting the origin and (X, Y) having values in $[0, 2\pi)$. Identifiy the distributions of R^2 and A.