STA213 **Fall 1999**

Homework 4

Suggested problems :

p246 : 5,8   p251 : 7   p257 : 11,12,13   p262 : 1,2
p272 : 1,7,8,12   p281 : 1,8   p292 : 9,10,17   p392 : 5,6

Problems for grading :

1. Let X, Y be independent and $X \sim G(\alpha_1, \beta)$ and $Y \sim G(\alpha_2, \beta)$.

(a) Show $X + Y$ and X/Y are independent.

(b) Show $X + Y$ and $X/(X + Y)$ are independent.

(c) Show $X + Y \sim G(\alpha_1 + \alpha_2, \beta)$ and $X/(X + Y) \sim Beta(\alpha_1, \alpha_2)$.

2. (t_n distribution)

(a) Derive the density of t_n distribution.

(b) Let $X \sim t_n$. Show $E(X) = 0$ and $Var(X) = n/(n-2)$ for $n > 2$.

3. ($F_{n,m}$ distribution)

(a) Derive the density of $F_{n,m}$ distribution.

(b) Let $X \sim F_{n,m}$. Show

$$\begin{aligned} E(X) &= n/(n-2), \text{ for } n > 2 \\ Var(X) &= \frac{n^2(2m + 2m - 4)}{m(n-2)^2(n-4)}, \text{ for } n > 4. \end{aligned}$$

(c) Show that if $X \sim F_{n,m}$, then

$$\frac{1}{1 + \frac{m}{n}X} \sim Beta(\frac{n}{2}, \frac{m}{2}).$$