# Suggested problems 2

## STA 213

These are the solutions only for the odd numbered problems. For the even numbered problems look on DeGroot.

**page 101**

1. $c = \frac{1}{15}$

5. $\Pr(X < 6) = 0.151$

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3. (a) $c = \frac{3}{7}$; (b) $\Pr(X > \frac{3}{2}) = \frac{37}{56}$

7. (a) $c = 2$; (b) $\Pr(1 < X < 2) = e^{-2} - e^{-4}$

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3. (a) 0.1; (b) 0.1; (c) 0.2; (d) 0; (e) 0.6; (f) 0.4; (g) 0.7; (h) 0; (i) 0; (j) 0; (k) 0; (l) 0.2

7.

$$F(z) = \begin{cases} 0 & \text{for } z < 0 \\ z^2 & \text{for } 0 \leq z \leq 1 \\ 1 & \text{for} z > 1 \end{cases}$$

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1. (a) 0.27; (b) 0.53; (c) 0.69; (d) 0.30; (e) 0.25

5. (a)

$$f(x,y) = \begin{cases} \frac{1}{2} & \text{for } (x,y) \in S \\ 0 & \text{otherwise} \end{cases}$$

(b) $\frac{\alpha}{2}$

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1. (a) $f_1(0) = \frac{3}{15}$; $f_1(1) = \frac{5}{15}$; $f_1(2) = \frac{7}{15}$; $f_2(0) = \frac{1}{10}$; $f_2(1) = \frac{2}{10}$; $f_2(2) = \frac{3}{10}$; $f_2(3) = \frac{4}{10}$; (b) No.

3. (a)

$$f_1(x) = \begin{cases} \frac{15}{4}x^2(1-x^2) & \text{for } -1 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$f_2(y) = \begin{cases} \frac{5}{2}(1-y)^{3/2} & \text{for } 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

(b) No.

7. No.

**page 132**

1. (a) $\frac{4}{33}$; (b) $\frac{10}{27}$ based only on the available information.

7. (a) 0.264 (b) 0.284 (c) 0.314

9.

$$g(x) = \begin{cases} 12x^2(1-x) & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

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1. (a) $\frac{1}{3}$

(b)

$$f_{12}(x_1,x_2) = \begin{cases} \frac{1}{3}(x_1 + 2x_2 + \frac{3}{2}) & \text{for } 0 \le x_i \le 1,\ i = 1,2 \\ 0 & \text{otherwise} \end{cases}$$

(c) $\frac{5}{13}$

3. (a) $\frac{1}{6}\pi$ (b) $\frac{1}{6}\pi$

**page 157**

1. $g(y) = \frac{2}{7}$ for $y = 0, 2, 6$, $g(12) = \frac{1}{7}$

7.

$$g(y) = \begin{cases} 2ye^{-y^2} & \text{for } y > 0 \\ 0 & \text{for } y \leq 0 \end{cases}$$

**page 169**

1.

$$g(y_1, y_2, y_3) = \begin{cases} \frac{8y_3}{y_1 y_2} & \text{for } 0 < y_3 < y_2 < y_1 < 1 \\ 0 & \text{otherwise} \end{cases}$$

3.

$$g(y) = \begin{cases} 2(1-y) & \text{for } 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

5.

$$g(z) = \begin{cases} z^2 & \text{for } 0 \leq z \leq 1 \\ z(2-z) & 1 < z \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

7. $n \geq \frac{\log 0.05}{\log 0.99}$

13.

$$f(y) = \begin{cases} n(n-1)y^{n-2}(1-y) & \text{for } 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

15. Since we can factor $f(x_1, x_2, \ldots, x_5) = g(x_1, x_2)h(x_3, x_4, x_5)$, this implies that there exists a $c$ such that the marginal of $X_1, X_2$ and the marginsal of $X_3, X_4, X5$ are

$$\begin{aligned} f_{12}(x_1, x_2) &= cg(x_1, x_2) \qquad (x_1, x_2) \in R^2 \\ f_{345}(x_3, x_4, x_5) &= \frac{1}{c}h(x_3, x_4, x_5) \qquad (x_3, x_4, x_5) \in R^3 \end{aligned}$$

Then $f(x_1, x_2, \ldots, x_5) = g(x_1, x_2)h(x_3, x_4, x_5)$. For any sets of real numbers $A_1, A_2$

$$\begin{aligned}
\Pr(Y_1 \in A_1, Y_2 \in A_2) &= \int_{r_1(x_1,x_2)\in A_1, r_2(x_3,x_4,x_5)\in A_2} \cdots \int f(x_1, x_2, \ldots, x_5)dx_1 \cdots dx_5 \\
&= \iint_{r_1(x_1,x_2)\in A_1} f_{12}(x_1, x_2)dx_1dx_2 \iiint_{r_2(x_3,x_4,x_5)\in A_2} f_{345}(x_3, x_4, x_5)dx_3dx_4dx_5 \\
&= \Pr(Y_1 \in A_1)\Pr(Y_2 \in A_2)
\end{aligned}$$

So $Y_1$ amd $Y_2$ are independent.