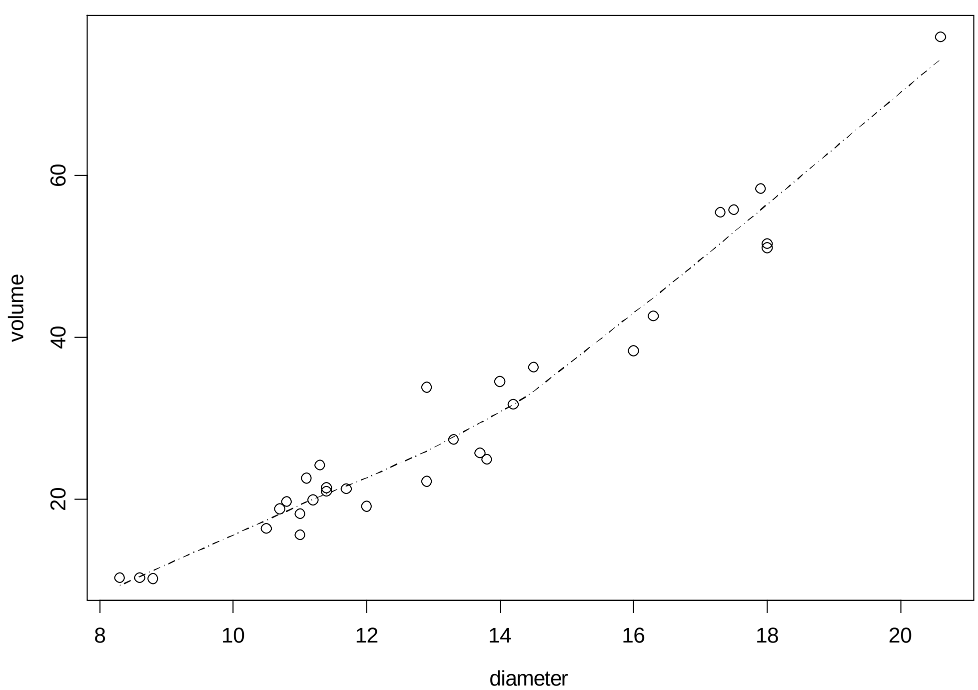
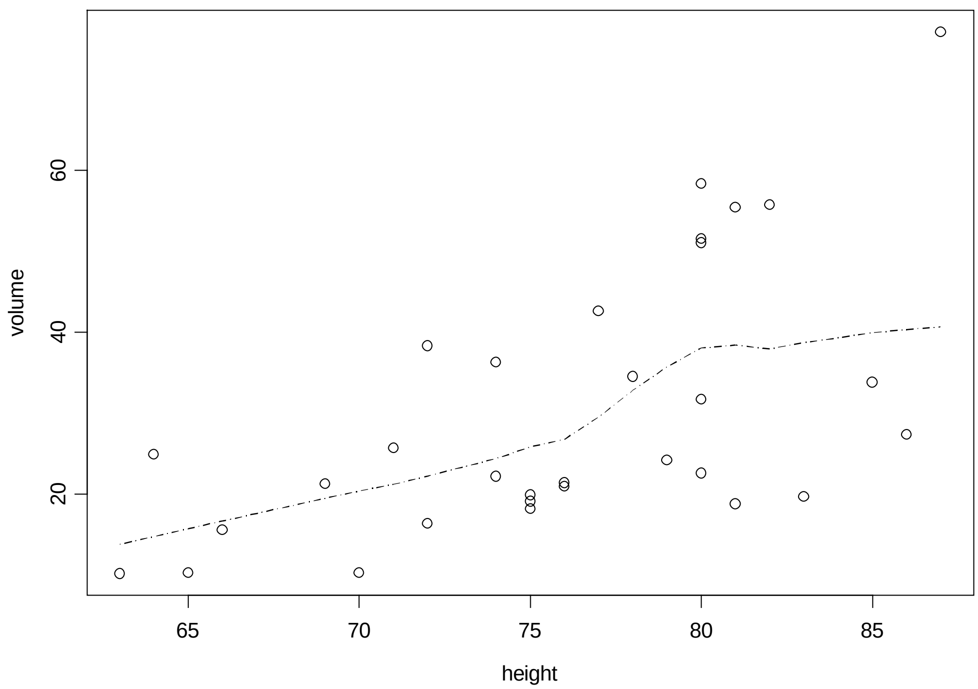
Trees: The volume of usable wood (volume) in cubic feet is given 31 black cherry trees, along with height (feet) and dameter (inches).
Volume of usable wood is the dependent variable.

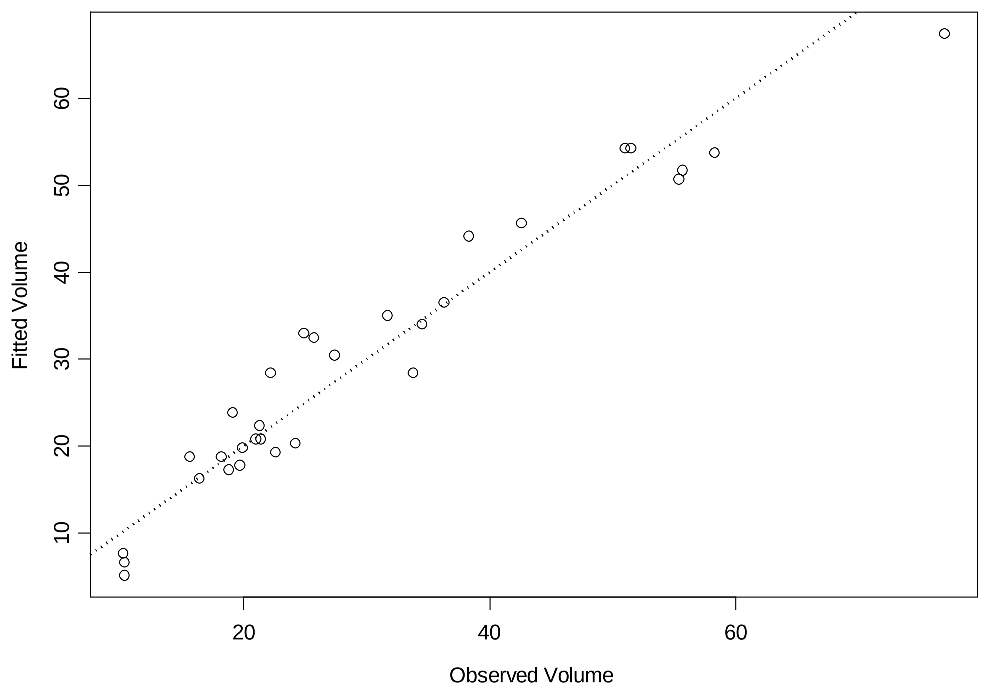
```
> trees <- read.table(paste(MyDir, "trees.dat", sep=""), header=T)
> attach(trees, pos=1)

> plot(diameter, volume)
> lines(lowess(diameter, volume), lty=3)
```



```
> plot(height, volume)
> lines(lowess(height, volume), lty=3)
```



```
> fit1 <- glm(volume ~ diameter,
              family=gaussian,
              link=identity)
> summary(fit1)

Call: glm(formula = volume ~ diameter,
family = gaussian, link = identity)
Deviance Residuals:
    Min      1Q  Median     3Q    Max
 -8.065  -3.107   0.152  3.495  9.587

Coefficients:
              Value Std. Error  t value
(Intercept) -36.943      3.365  -10.978
   diameter   5.066      0.247   20.478

(Dispersion Parameter for Gaussian family
taken to be 18.079 )

    Null Deviance: 8106.084 on 30 degrees
of freedom

Residual Deviance: 524.3025 on 29 degrees
of freedom

Number of Fisher Scoring Iterations: 1

Correlation of Coefficients:
         (Intercept)
diameter -0.9739092
```


Fitted Volume
10
20
30
40
50
60
20
40
60
Observed Volume

```
> fit1 <- glm(volume ~ diameter,
          family=gaussian, link=identity)
>
> names(fit1)
 [1] "coefficients"      "residuals"
 [3] "fitted.values"     "effects"
 [5] "R"                 "rank"
 [7] "assign"            "df.residual"
 [9] "weights"           "family"
[11] "linear.predictors" "deviance"
[13] "null.deviance"     "call"
[15] "iter"              "y"
[17] "contrasts"         "terms"
[19] "formula"           "control"

> glm(volume ~ volume, family=gaussian)
Call:
glm(formula = volume ~ volume, family =
gaussian)

Coefficients:
 (Intercept)
    30.17097

Degrees of Freedom: 31 Total; 30 Residual
Residual Deviance: 8106.084

> fit1$deviance
[1] 524.3025
> fit1$null.deviance
[1] 8106.084
```

```
> 1-fit1$deviance/fit1$null.deviance
[1] 0.9353199
```

= R^2

What about nonlinear relationship?

Box-Cox transformation (y>0):

$$y(\lambda) = \begin{cases} \dfrac{y^\lambda - 1}{\lambda} & \lambda \neq 0 \\ \log(y) & \lambda = 0 \end{cases}$$

Treat $y_i(\lambda) \sim N\left(X_i\beta, \sigma^2\right)$, for some λ.

Solve over grid:

- Fix λ,
- compute likelihood,
- pick maximum.

Density (likelihood) of y_i if $z_i = \frac{y_i^\lambda - 1}{\lambda} \sim N\left(\mu_z, \sigma_z^2\right)$.

$$f(y) = f(z)\left|\frac{dz}{dy}\right| \text{ and } \frac{dz}{dy} = \frac{d}{dy}\left(\frac{y^\lambda - 1}{\lambda}\right) = y^{\lambda-1}$$

So,

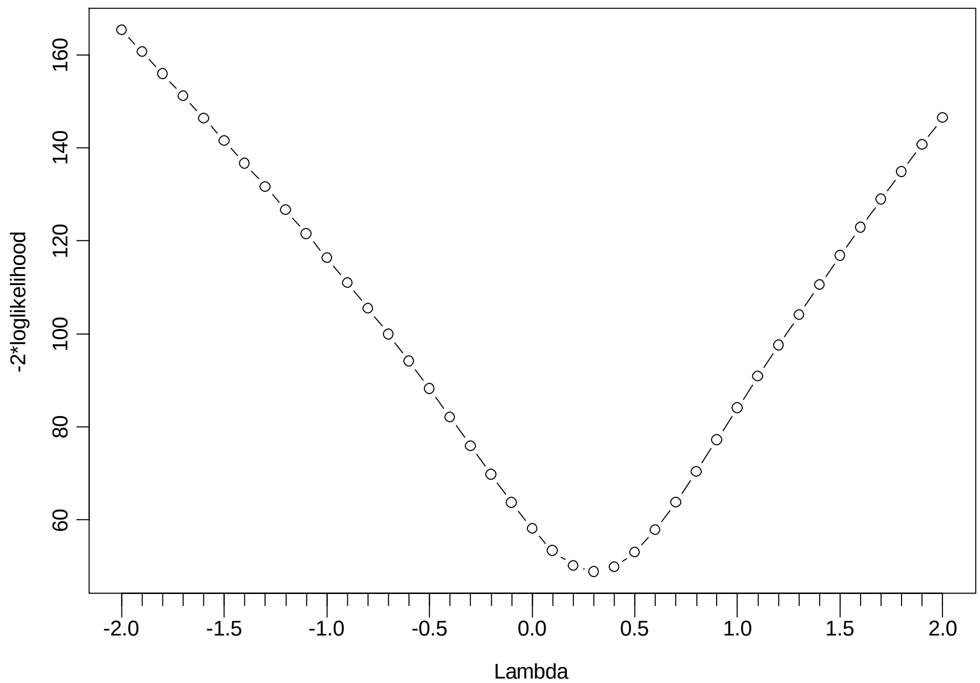
$$f(y) = f(z)\left|\frac{dz}{dy}\right| = f(z)y^{\lambda-1}$$

$$= \frac{y^{\lambda-1}}{\sqrt{2\pi\sigma_z^2}} \exp\left(\frac{-\left(\frac{y^\lambda - 1}{\lambda} - \mu_z\right)^2}{2\sigma_z^2}\right)$$

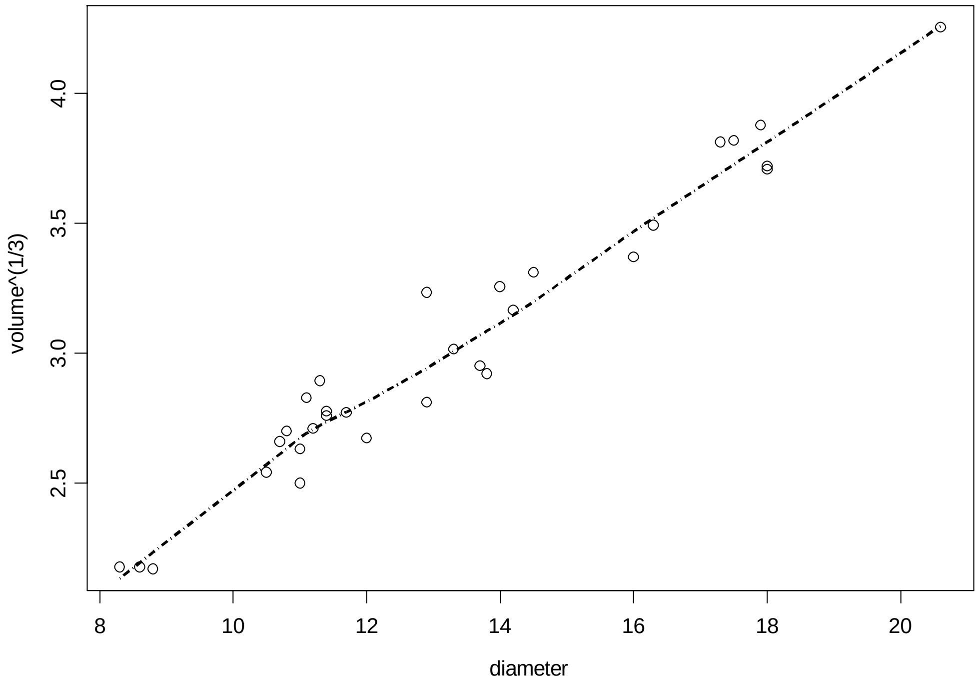
S functions:

```
BoxCox <- function(y, lambda)
   if(lambda == 0) log(y) else
      (y^lambda - 1)/lambda

BoxCoxLike <- function(Lambda=0, GLMfit,
OrigY) {
  RSS <- deviance(GLMfit)
  n <- length(GLMfit$y)
  sigma.hat <-
     deviance(GLMfit)/GLMfit$df.residual
  Logli <- (Lambda-1)*sum(log(OrigY)) –
            n*log(sqrt(sigma.hat))
-2*Logli
}

lambda <- seq(from=-2, to=2, by=0.1)
logli.vec <- rep(NA, length(lambda))
likeli.vec <- logli.vec
for (i in 1:length(lambda)) {
  l <- lambda[i]
  y.l <- BoxCox(volume, l)
  fit <- glm(y.l ~ diameter+height,
             family=gaussian,
             link=identity)
  logli.vec[i] <- BoxCoxLike(Lambda=l,
                  GLMfit=fit, OrigY=y)
  likeli.vec[i] <- exp(-logli.vec[i]/2)
}
```

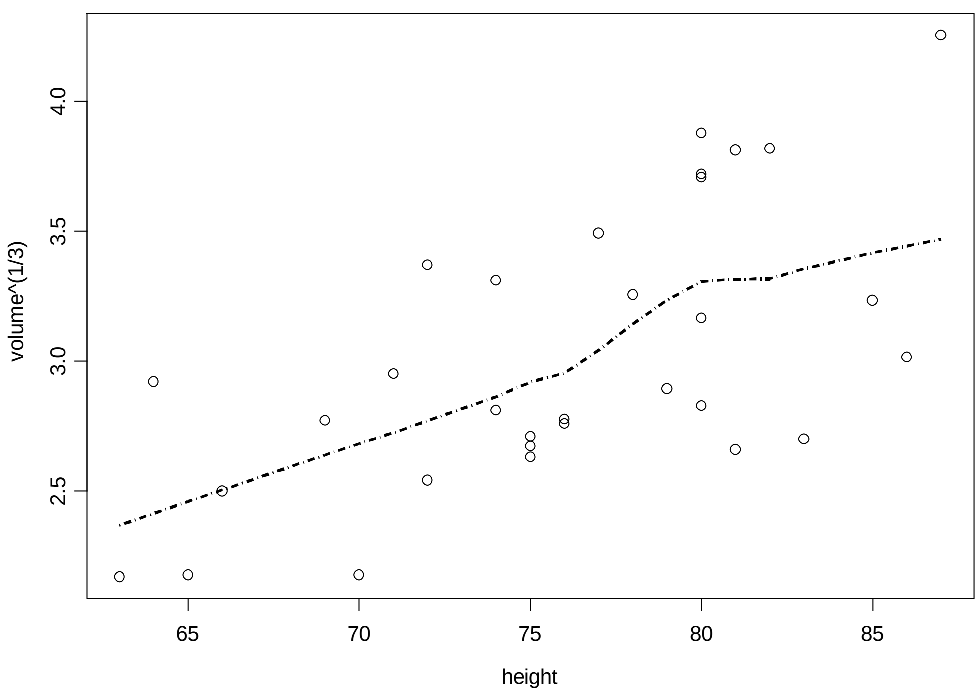
```
plot(lambda, logli.vec, type="b")
```



Diameter



Height



```
> fit.cube <- glm(y.cuberoot ~
  diameter+height, family=gaussian)
> summary(fit.cube)

Call: glm(formula = y.cuberoot ~ diameter +
height, family = gaussian)
Deviance Residuals:
     Min        1Q   Median        3Q    Max
 -0.1596  -0.0502  -0.0068  0.0696  0.1340

Coefficients:
              Value  Std. Error   t value
(Intercept) -0.0854      0.1843   -0.4633
   diameter  0.1515      0.0056   26.8711
     height  0.0145      0.0028    5.2112

(Dispersion Parameter for Gaussian family
taken to be 0.0068603 )

    Null Deviance: 8.596 on 30 deg. of freedom

Residual Deviance: 0.192 on 28 deg. of freedom

Number of Fisher Scoring Iterations: 1

Correlation of Coefficients:
         (Intercept)   diameter
diameter     0.189
  height    -0.935      -0.519
```

$R^2 = 0.978$

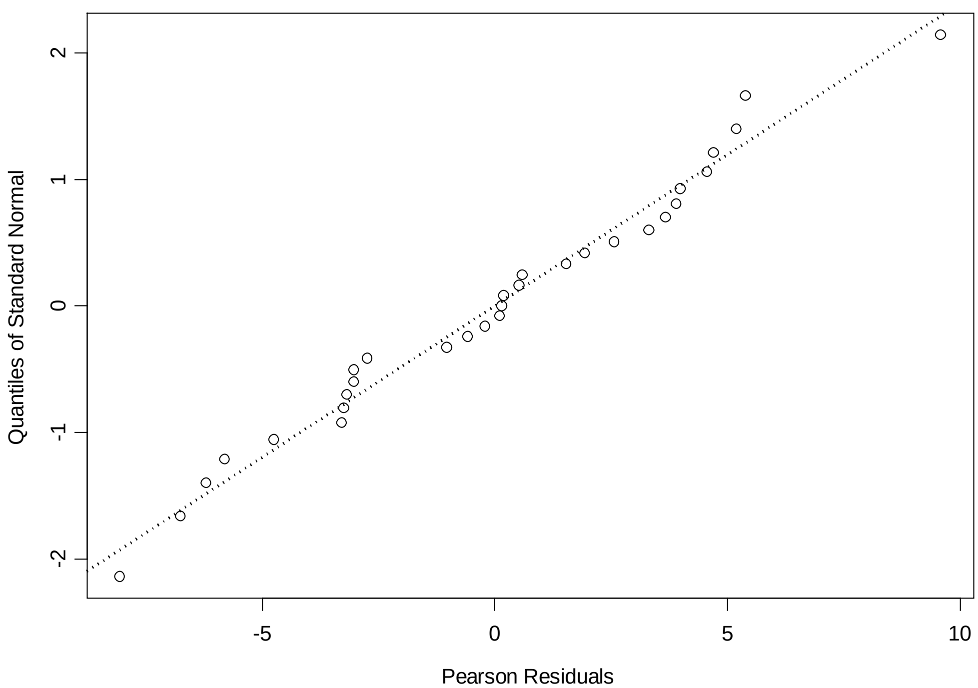
Residuals:

Pearson Residuals:

$$r_i^P = \frac{y_i - \hat{\mu}_i}{\sqrt{V(\hat{\mu}_i)}}$$

$$\sum_{i=1}^{n} \left(r_i^P\right)^2 = \chi^2 \text{ statistic}$$

```
> Pears.Resid <- resid(fit1, type="pearson")
> qqnorm(Pears.Resid, datax=T, xlab="Pearson Residuals")
> abline(0, 1/sqrt(var(Pears.Resid)), lty=2, lwd=3)
```



Deviance Residuals:

$$r_i^D = sign(y_i - \hat{\mu}_i)d_i$$

$$d_i = \text{contribution of i}^{\text{th}} \text{ obs'n}$$

$$\text{to deviance, } D.$$

$$\sum_{i=1}^{n}\left(r_i^D\right)^2 = \text{Deviance statistic}$$

Deviance (scaled):

$$D(c, f) = -2\log\left(\frac{\mathrm{L_c}}{\mathrm{L_f}}\right)$$

$$= 2\sum\left[y_i\left(\hat{\theta}_i - \tilde{\theta}_i\right) + b\left(\tilde{\theta}_i\right) - b\left(\hat{\theta}_i\right)\right] / a_i(\phi)$$

where "~" denotes current model and "^" denotes the saturated model.

If $a_{i(\phi)} = \phi / w_i$

$$D(c, f) = 2\sum w_i\left[y_i\left(\hat{\theta}_i - \tilde{\theta}_i\right) + b\left(\tilde{\theta}_i\right) - b\left(\hat{\theta}_i\right)\right] / \phi$$

$$= \frac{D_U(c, f)}{\phi}$$

GLM Deviance = $D_U(c, f) = \phi\, D(c, f)$

For Gaussian distribution,

$$D(c, f) = 2\sum w_i \left[y_i\left(\hat{\theta}_i - \tilde{\theta}_i\right) + b\left(\tilde{\theta}_i\right) - b\left(\hat{\theta}_i\right)\right] / \phi$$

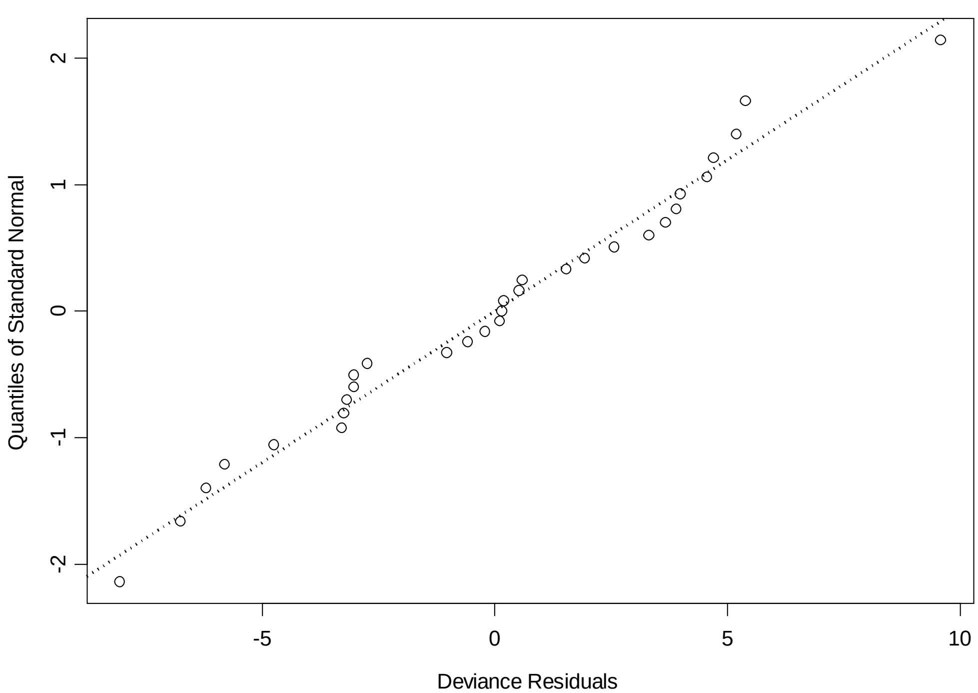
$$= 2\sum 1\left[y_i\left(y_i - X_i\hat{\beta}\right) + \frac{\left(X_i\hat{\beta}\right)^2}{2} - \frac{\left(y_i\right)^2}{2}\right] \Big/ \sigma^2$$

$$= \frac{\sum\left(y_i - X_i\hat{\beta}\right)^2}{\sigma^2}$$

So,

$$D_U(c, f) = \phi\, D(c, f) = \sum\left(y_i - X_i\hat{\beta}\right)^2$$

```
> Dev.Resid <- resid(fit1, type="deviance")
> qqnorm(Dev.Resid, datax=T, xlab="Deviance Residuals")
> abline(0, 1/sqrt(var(Dev.Resid)), lty=2, lwd=3)
```



Working Residuals

$$r_i^W = (y_i - \hat{\mu}_i)\frac{d\hat{\eta}_i}{d\hat{\mu}_i}$$

If Gaussian dist'n,

$$\eta_i = \mathrm{X}_i\beta = \mu_\mathrm{i}$$

so,

$$\frac{d\hat{\eta}_i}{d\hat{\mu}_i} = 1 \text{ and}$$

$$r_i^W = (y_i - \hat{\mu}_\mathrm{i})$$

Response Residuals:

$$r_i^R = (y_i - \hat{\mu}_\mathrm{i})$$

For Gaussian family, all 4 residuals are the same!

Residuals after cube-root transformation