STA 110B Fall 1997

Final Exam
December 15, 1997

Name:

Section:

I understand and agree to abide by the Duke honor code,

Signed:

Instructions

This is a closed-book exam, however, one 8.5 by 11 inch "crib sheet" is permitted. You may use a calculator if you find it useful. Show your work in the space provided, but be concise. Correct but unsubstantiated answers will receive no credit.

Point assignments for each of the problems are given in parentheses in the table below. You have 3 hours total; plan accordingly. You must hand the exam at or before Noon, no extra time will be given. Good luck!

		Page 1	Page 2	Page 3
1.	(50)			
2.	(50)			
Total	(100)			

1) A researcher, interested in the relationship between a variable Y and a covariable X, collects a random sample (n=122) from the population of interest. The researcher's hypothesis is that Y and X are linearly unrelated. To test this hypothesis he fits the regression model

$$Y_i = \alpha + \beta X_i + \epsilon_i$$

and obtains the estimates $b = 0.0200$ of β with standard error $s.e.(b) = 0.0100$, and $a = 1.50$ of α with standard error $s.e.(a) = 1.25$.

The following multiple choice questions relate to this problem, **mark correct answer(s) with an X**:

a) (**4 points**) The researcher's hypothesis can be written

☐ *i)* $\mathrm{H}_o : b = 0$, ☐ *ii)* $\mathrm{H}_A : \beta \neq 0$, ☐ *iii)* $\mathrm{H}_o : \beta = 0$,

☐ *iv)* $\mathrm{H}_o : \alpha = 0$, ☐ *v)* $\mathrm{H}_A : \alpha \neq 0$, ☐ *vi)* $\mathrm{H}_o : a = 0$.

b) (**4 points**) The t-statistic for a test of the researcher's hypothesis is

☐ *i)* $t = 0.02$, ☐ *ii)* $t = 0.20$, ☐ *iii)* $t = 2.00$,

which has

☐ *i)* $df = 120$, ☐ *ii)* $df = 121$, ☐ *iii)* $df = 122$.

degrees of freedom associated with it.

The researcher calculates the p-value of his test to be 0.048.

c) (**5 points**) The p-value is defined to be the probability

☐ *i)* that the null hypothesis is true,

☐ *ii)* of observing as (or more) extreme a t-statistic assuming the null hypothesis is true.

1) Continued. A researcher, interested in the relationship between a variable Y and a covariable X, collects a random sample (n=122) from the population of interest. The researcher's hypothesis is that Y and X are linearly unrelated. To test this hypothesis he fits the regression model

$$Y_i = \alpha + \beta X_i + \epsilon_i$$

and obtains the estimates $b = 0.0200$ of β with standard error $s.e.(b) = 0.0100$, and $a = 1.50$ of α with standard error $s.e.(a) = 1.25$.

The following multiple choice questions relate to this problem, **mark correct answer(s) with an X**:

d) (**4 points**) The researcher can conclude that

☐ *i)* a linear relationship between X and Y is discernible at the 10% level,

☐ *ii)* a linear relationship between X and Y is discernible at the 5% level,

☐ *iii)* a linear relationship between X and Y is discernible at the 2% level,

☐ *iv)* a linear relationship between X and Y is discernible at the 1% level.

e) (**4 points**) The "level" referred to in part **e)**, is the probability of

☐ *i)* type I error, α, ☐ *ii)* type II error, β,

and is set by the researcher

☐ *i)* in advance of the experiment. ☐ *ii)* after the sample is taken.

f) (**5 points**) Another way for the researcher to conduct such a classical hypothesis test, without the use of p-values, is to calculate the t-statistic and reject the null hypothesis if it is

☐ *i)* more extreme than its critical value ☐ *ii)* less extreme than its critical value

g) (**4 points**) By drawing a larger sample, the researcher would have

☐ *i)* reduced ☐ *ii)* increased

the probability of committing a type II error. A type II error is defined as

☐ *i)* rejecting H_o when it is true. ☐ *ii)* failing to reject H_o when it is false.

1) Continued. A researcher, interested in the relationship between a variable Y and a covariable X, collects a random sample (n=122) from the population of interest. The researcher's hypothesis is that Y and X are linearly unrelated. To test this hypothesis he fits the regression model

$$Y_i = \alpha + \beta X_i + \epsilon_i$$

and obtains the estimates $b = 0.0200$ of β with standard error $s.e.(b) = 0.0100$, and $a = 1.50$ of α with standard error $s.e.(a) = 1.25$.

h) (**5 points**) What value does the fitted regression predict for Y when $X = 3$?

i) (**5 points**) Calculate a 95% confidence interval for the population slope β

j) (**6 points**) A test of the researcher's hypothesis at the 5% level could also be conducted by

☐ *i)* rejecting the hypothesis ☐ *ii)* failing to reject the hypothesis

if the hypothesized

☐ *i)* estimate b ☐ *ii)* parameter β

falls outside the 95% confidence interval for

☐ *i)* the estimate b. ☐ *ii)* the parameter β.

k) (**4 points**) Using least squares, the researcher's estimate of the regression slope will be

☐ *i)* unbiased, ☐ *ii)* biased

if the regression model is correct.

2) Twenty-four subjects with good hearing were each given 4 different hearing exams in the presence of background noise. The purpose of this exercise was to determine if the 4 exams were calibrated equally on normal-hearing subjects before they were used to diagnose hearing ability. The variable of interest, `score`, is the score obtained on the exam. A two-way analysis of variance is used to model the variability in exam `score` as a function of two categorical variables: `subject` and `exam`. By including the variable `subject` in the model we are controlling for variability between subjects. Hence, any effect ascribed to `exam` should be due to differences in calibration between the four exams. The ANOVA table obtained from fitting the two-way analysis of variance follows.

Hearing Test ANOVA Table

Source	Df	SS	MS	F	P
Exam	??	??????	?????	????	0.000042
Subject	23	3291.8	143.1	3.96	0.0000049
Error	69	2495.9	?????		
Total	95	6705.0			

a) (**5 points**) Calculate the sum of squares explained by `exam`.

b) (**5 points**) Calculate the mean square for the variable `exam`.

c) (**5 points**) Calculate the F-ratio for the test for a difference in calibration of hearing exams.

d) (**5 points**) How many degrees of freedom are associated with this F-ratio?

e) (**5 points**) Can the hypothesis that there is no difference in hearing exams be rejected at the $\alpha = 5\%$ level? Why/why not?

2) Hearing experiment, continued. The 2-way analysis of variance model for the hearing test data specifies that the expected hearing score for subject i on exam j is

$$\mu + \alpha_i + \beta_j,$$

where μ is the baseline score, α_i is the coefficient of the dummy variable $\texttt{Subjct}_i$ and β_j is the coefficient of the dummy variable $\texttt{Exam}_j$. Estimates of these coefficients, found using multiple linear regression with dummy variables to identify subjects (`Subjct1, ..., Subjct24`) and exams (`Exam1, ..., Exam4`), follows. Use it to answer the remaining questions. Note that Subject 9's performance on Exam 4 serves as the "reference group."

Summary of Fit

Mean of Response	28.3125	R-Square	0.6236
Root MSE	6.0272	Adj R-Sq	0.4817

Model Equation

HEARING = 33.7708 − 12.0000 P_2 − 6.0000 P_3 − 8.5000 P_4 − 5.5000 P_5 − 4.5000 P_6 − 0.5000 P_7 − 3.0000 P_8 − 9.5000 P_9 − 8.0000 P_10 − 17.0000 P_11 + 1.5000 P_12 − 12.5000 P_13 − 15.0000 P_14 − 19.5000 P_15 − 14.5000 P_16 − 9.5000 P_17 + 3.5000 P_18 − 8.5000 P_19 − 16.0000 P_20 − 5.5000 P_21 − 8.5000 P_22 − 9.5000 P_23 − 8.0000 P_24 + 7.1667 P_26 + 4.0833 P_27 − 0.3333 P_28

Parameter Estimates

Variable	SUBJECT	EXAM	DF	Estimate	Std Error	T Stat	Prob >|T|	Tolerance	Var Inflation
INTERCEP			1	33.7708	3.1964	10.5653	0.0001	.	0
SUBJECT	Subjct1		1	-12.0000	4.2618	-2.8157	0.0063	0.5217	1.9167
	Subjct10		1	-6.0000	4.2618	-1.4078	0.1637	0.5217	1.9167
	Subjct11		1	-8.5000	4.2618	-1.9944	0.0501	0.5217	1.9167
	Subjct12		1	-5.5000	4.2618	-1.2905	0.2012	0.5217	1.9167
	Subjct13		1	-4.5000	4.2618	-1.0559	0.2947	0.5217	1.9167
	Subjct14		1	-0.5000	4.2618	-0.1173	0.9069	0.5217	1.9167
	Subjct15		1	-3.0000	4.2618	-0.7039	0.4839	0.5217	1.9167
	Subjct16		1	-9.5000	4.2618	-2.2291	0.0291	0.5217	1.9167
	Subjct17		1	-8.0000	4.2618	-1.8771	0.0647	0.5217	1.9167
	Subjct18		1	-17.0000	4.2618	-3.9889	0.0002	0.5217	1.9167
	Subjct19		1	1.5000	4.2618	0.3520	0.7259	0.5217	1.9167
	Subjct2		1	-12.5000	4.2618	-2.9330	0.0046	0.5217	1.9167
	Subjct20		1	-15.0000	4.2618	-3.5196	0.0008	0.5217	1.9167
	Subjct21		1	-19.5000	4.2618	-4.5755	0.0001	0.5217	1.9167
	Subjct22		1	-14.5000	4.2618	-3.4023	0.0011	0.5217	1.9167
	Subjct23		1	-9.5000	4.2618	-2.2291	0.0291	0.5217	1.9167
	Subjct24		1	3.5000	4.2618	0.8212	0.4143	0.5217	1.9167
	Subjct3		1	-8.5000	4.2618	-1.9944	0.0501	0.5217	1.9167
	Subjct4		1	-16.0000	4.2618	-3.7542	0.0004	0.5217	1.9167
	Subjct5		1	-5.5000	4.2618	-1.2905	0.2012	0.5217	1.9167
	Subjct6		1	-8.5000	4.2618	-1.9944	0.0501	0.5217	1.9167
	Subjct7		1	-9.5000	4.2618	-2.2291	0.0291	0.5217	1.9167
	Subjct8		1	-8.0000	4.2618	-1.8771	0.0647	0.5217	1.9167
	Subjct9		0	0	.	.	.	.	.
EXAM		Exam1	1	7.1667	1.7399	4.1190	0.0001	0.6667	1.5000
		Exam2	1	4.0833	1.7399	2.3469	0.0218	0.6667	1.5000
		Exam3	1	-0.3333	1.7399	-0.1916	0.8486	0.6667	1.5000
		Exam4	0	0	.	.	.	.	.

2) Hearing experiment, continued.

f) (**4 points**) What is the predicted hearing score for Subject 1 on Exam 2?

g) (**4 points**) On average, how much higher/lower did Subject 1 score than Subject 9?

h) (**4 points**) On average, how much higher/lower do subjects score on Exam 2, than on Exam 4?

i) (**4 points**) Are hearing scores on Exam 2 discernibly different at the 5% level from those on Exam 4? Why/why not?

j) (**5 points**) Are hearing scores on Exam 2 discernibly different at the 1% level from those on Exam 4? Why/why not?

2) Hearing experiment, continued.

k) (**4 points**) Calculate a 95% confidence interval for the mean difference in scores between Exam 1 and Exam 4.