STA 110B | Name____________________

Spring 2000 | Section____________________

Quiz 10

week of 10APR2000

OLS estimates for slope and intercept: $b = \frac{\Sigma(X-\bar{X})(Y-\bar{Y})}{\Sigma(X-\bar{X})^2}$, $a = \bar{Y} - b\bar{X}$. Also, $SE_b = \frac{s}{\sqrt{\Sigma(X-\bar{X})^2}}$.
95% confidence interval for the mean (of Y_0) at level X_0:

$$(a + bX_0) \pm t^{n-2}_{.025} s \sqrt{\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\Sigma(X - \bar{X})^2}}$$

95% prediction/confidence interval for an individual Y_0 at level X_0:

$$(a + bX_0) \pm t^{n-2}_{.025} s \sqrt{\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\Sigma(X - \bar{X})^2} + 1}$$

1. Auditors often are required to compare the audited (or current) value Y of an inventory item with the book (or listed) value X. If a company is keeping its inventory and books up to date, there should be a strong linear relationship between these values. A sample of 10 inventory items from a certain company gave the following data. (We are interested in the model $Y = \alpha + \beta X + e$.)

$\bar{Y} = 72.1$ $\quad\quad$ $\bar{X} = 72$ $\quad\quad$ $\Sigma(X - \bar{X})^2 = 54714$

$\Sigma(X - \bar{X})(Y - \bar{Y}) = 54243$ $\quad\quad$ $s^2 = \frac{1}{n-2}\Sigma(Y - \hat{Y})^2 = 7.10568$

a. (2 points) Estimate the expected change in audited value for a 1-unit change in book value.

$$b = \frac{\Sigma(X - \bar{X})(Y - \bar{Y})}{\Sigma(X - \bar{X})^2} = \frac{54243}{54714} \approx 0.9914$$

b. (3 points) If the book value is 100, what is your best estimate for the audited value?

$$a = \bar{Y} - b\bar{X} = 72.1 - \frac{54243}{54714}(72) \approx 0.7198$$

If you use the rounded estimate for b from part (a) above, you will get $a \approx 0.7192$. This is fine, but just be aware that where you round can influence your answer slightly.
So, your best estimate is $a + b(100) \approx 0.7198 + (0.9914)(100) \approx 99.86$.

c. (2 points) Find a 90% confidence interval for β.

$$\begin{array}{rcl} b & \pm & t^{n-2}_{.05}\sqrt{\dfrac{s^2}{\Sigma(X - \bar{X})^2}} \\ 0.9914 & \pm & 1.86\sqrt{\dfrac{7.10568}{54714}} \\ 0.9914 & \pm & 0.0212 \\ (0.9702 & , & 1.0126) \end{array}$$

d. (3 points) Give a 90% interval for the average audit value if the book value is 100.

$$
\begin{aligned}
(a + bX_0) &\pm t^{n-2}_{.025}\, s\sqrt{\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\Sigma(X - \bar{X})^2}} \\
99.859 &\pm 1.86\sqrt{7.10568}\sqrt{\frac{1}{10} + \frac{(100 - 72)^2}{54714}} \\
99.859 &\pm 1.6765 \\
(98.183 &, 101.54)
\end{aligned}
$$