STA 110B Name_____

Spring 2000 Section_____

Quiz 3

week of 07FEB2000

1. (4 points) As items come to the end of a production line, an inspector chooses which items are to go through a complete inspection. Ten percent of all items produced are defective. Sixty percent of all defective items go through a complete inspection, and 20% of all good items go through a complete inspection. Given that an item is completely inspected, what is the probability it is defective?

D: item is defective
I: item gets complete inspection

$$P(D) = 0.10$$
$$P(I|D) = 0.60$$
$$P(I|\bar{D}) = 0.20$$

$$\begin{aligned} P(D|I) &= \frac{P(I|D)P(D)}{P(I)} \\ &= \frac{P(I|D)P(D)}{P(I|D)P(D) + P(I|\bar{D})P(\bar{D})} \\ &= \frac{(0.60)(0.10)}{(0.60)(0.10) + (0.20)(0.90)} \\ &= \frac{0.06}{0.06 + 0.18} \\ &= 0.25 \end{aligned}$$

2. A large group of people is to be checked for two common symptoms of a certain disease. It is thought that 20% of the people possess symptom A alone, 30% posseess symptom B alone, 10% possess both symptoms, and the remainder have neither symptom. For one person chosen at random from this group, find these probabilities:

a. (2 points) that the person has neither symptom

$$\begin{aligned} P(A \cap \bar{B}) &= 0.20 \\ P(\bar{A} \cap B) &= 0.30 \\ P(A \cap B) &= 0.10 \end{aligned}$$

$$\begin{aligned} P(\bar{A} \cap \bar{B}) &= 1 - P(A \cup B) \\ P(\bar{A} \cap \bar{B}) &= 1 - [P(A \cap \bar{B}) + P(\bar{A} \cap B) + P(A \cap B)] \\ &= 1 - (0.20 + 0.30 + 0.10) \\ &= 1 - 0.60 \\ &= 0.40 \end{aligned}$$

b. (2 points) that the person has at least one symptom

$$\begin{aligned} P(A \cup B) &= P(A \cap \bar{B}) + P(\bar{A} \cap B) + P(A \cap B) \\ &= 0.20 + 0.30 + 10 \\ &= 0.60 \end{aligned}$$

or

$$\begin{aligned} P(A \cup B) &= 1 - P(\bar{A} \cap \bar{B}) \\ &= 1 - 0.40 \\ &= 0.60 \end{aligned}$$

c. (2 points) that the person has both symptoms, given that he has symptom B

$$\begin{aligned} P(A|B) &= \frac{P(A \cap B)}{P(B)} \\ &= \frac{P(A \cap B)}{P(A \cap B) + P(\bar{A} \cap B)} \\ &= \frac{0.10}{0.10 + 0.30} \\ &= \frac{0.10}{0.40} \\ &= 0.25 \end{aligned}$$