STA 110B	Name________________
Spring 2000	Section________________

Quiz 5

week of 21FEB2000

Contracts for two construction jobs are randomly asssigned to one or more of three firms, A, B, and C. Let Y_1 denote the number of contracts assigned to firm A and Y_2 the number of contracts assigned to firm B. Recall that each firm can receive 0, 1, or 2 contracts. The joint probability distribution $p(y_1, y_2)$ is given below.

	Y_1		
Y_2	0	1	2
0	1/9	2/9	1/9
1	2/9	2/9	0
2	1/9	0	0

You might want to use the following mathematical relationships:

$$\begin{aligned} Var(X) &= E[(X - E(X))^2] \\ Cov(X, Y) &= E[(X - E(X))(Y - E(Y))] \end{aligned}$$

1. (2 points) Find the marginal probability distribution of Y_1.

y_1	$p(y_1)$
0	4/9
1	4/9
2	1/9

2. (2 points) Find the expected value of Y_1.

$$\begin{aligned} E(Y_1) &= \Sigma y_1 p(y_1) \\ &= (0)\frac{4}{9} + (1)\frac{4}{9} + (2)\frac{1}{9} \\ &= \frac{4}{9} + \frac{2}{9} \\ &= \frac{6}{9} \end{aligned}$$

3. (2 points) Find the covariance Cov(Y_1, Y_2) between Y_1 and Y_2.

$$\begin{aligned} Cov(Y_1, Y_2) &= E[(Y_1 - E(Y_1))(Y_2 - E(Y_2))] \\ &= E(Y_1 Y_2) - E(Y_1)E(Y_2) - E(Y_1)E(Y_2) + E(Y_1)E(Y_2) \\ &= E(Y_1 Y_2) - E(Y_1)E(Y_2) \end{aligned}$$

$$
\begin{aligned}
E(Y_1Y_2) &= \Sigma y_1 y_2 p(y_1, y_2) \\
&= \frac{2}{9}
\end{aligned}
$$

$$
\begin{aligned}
Cov(Y_1, Y_2) &= E(Y_1Y_2) - E(Y_1)E(Y_2) \\
&= \frac{2}{9} - (\frac{6}{9})(\frac{6}{9}) \\
&= \frac{18}{81} - \frac{36}{81} \\
&= -\frac{18}{81} \\
&= -\frac{2}{9}
\end{aligned}
$$

4. (2 points) Are Y_1 and Y_2 independent? Why or why not?

No, Y_1 and Y_2 are not independent. If Y_1 and Y_2 were independent, $Cov(Y_1, Y_2)$ would be 0. Or, see if $p(y_1, y_2) = p_{Y_1}(y_1)p_{Y_2}(y_2)$.

$$
\begin{aligned}
p_{Y_1}(0) &= \frac{4}{9} \\
p_{Y_2}(0) &= \frac{4}{9} \\
p(0,0) &= \frac{1}{9}
\end{aligned}
$$

Since $p(0,0) = \frac{1}{9}$ does not equal $p_{Y_1}(0)p_{Y_2}(0) = \frac{16}{81}$, Y_1 and Y_2 are not independent.

5. (2 points) Let Z equal the total amount to be paid to complete these contracts. Firm A charges \$9000; firm B charges \$12000. Therefore, we can express costs as $Z = 9000Y_1 + 12000Y_2$, where Z is measured in dollars. Find the expected cost, $E(Z)$.

$$
\begin{aligned}
E(Z) &= E(9000Y_1 + 12000Y_2) \\
&= 9000E(Y_1) + 12000E(Y_2) \\
&= (9000)(\frac{6}{9}) + (12000)(\frac{6}{9}) \\
&= 6000 + 8000 \\
&= 14000
\end{aligned}
$$

OR

$$
\begin{aligned}
E(Z) &= (\frac{2}{9})(9000) + (\frac{1}{9})(18000) + (\frac{2}{9})(12000) + (\frac{2}{9})(21000) + (\frac{1}{9})(24000) \\
&= 2000 + 2000 + \frac{24000}{9} + \frac{42000}{9} + \frac{24000}{9} \\
&= 4000 + 10000 \\
&= 14000
\end{aligned}
$$