STA 110B Name____________________

Spring 2000 Section____________________

Quiz 8

week of 27MAR2000

Remember: The standard error for the difference in proportions is $\sqrt{\frac{P_1(1-P_1)}{n_1}+\frac{P_2(1-P_2)}{n_2}}$.

1. A consumer agency wants to determine whether there is a difference between the proportions of the 2 leading automobile models that need major repairs (more than \$500) within 2 years of their purchase. Of a random sample of 400 2-year owners of model 1, 53 reported that their cars need major repairs. For the random sample of 500 2-year model 2 owners this number was 78.

a. (2 points) Construct a two-sided 95% confidence interval for the proportion of model 1 cars needing major repairs minus the proportion of model 2 cars needing major repairs $\pi_1 - \pi_2$.

$$n_1 = 400, P_1 = \frac{53}{400} = 0.1325$$

$$n_2 = 500, P_2 = \frac{78}{500} = 0.156$$

$$\begin{array}{rcl} (P_1 - P_2) & \pm & z_{.025}\sqrt{\frac{P_1(1-P_1)}{n_1}+\frac{P_2(1-P_2)}{n_2}} \\ (0.1325 - 0.156) & \pm & 1.96\sqrt{\frac{0.1325(0.8675)}{400}+\frac{0.156(0.844)}{500}} \\ (-0.069 & , & 0.022) \end{array}$$

b. (1 point) If we're interested in whether or not a discernible difference exists between the proportions needing major repairs, define the appropriate hypotheses in symbols and in words.

H_0: $\pi_1 - \pi_2 = 0$ proportions of the 2 types of cars needing major repairs within 2 years are the same
H_A: $\pi_1 - \pi_2 \neq 0$ proportions of the 2 types of cars needing major repairs within 2 years are different

c. (3 points) At the 5% error level, are the proportions needing major repairs within 2 years discernably different? Why or why not?

Since 0 falls within the confidence interval given in part (a), the proportions are not discernibly different at the 5% error level.

d. (1 point) If we're now interested in knowing whether the percentage of model 2 cars that need major repairs in the first 2 years is discernibly *higher* than the percentage of model 1 cars that need major repairs, what are the hypotheses (use both symbols and words)?

H_0: $\pi_1 - \pi_2 = 0$ proportions of the 2 types of cars needing major repairs within 2 years are the same

H_A: $\pi_1 - \pi_2 < 0$ proportion of model 2 cars needing major repairs within 2 years is higher than that of model 1 cars

e. (3 points) At the 5% error level, is the proportion of model 2 cars needing major repairs within 2 years discernibly *higher* than the percentage of model 1 cars that need major repairs?

Test statistic:

$$Z = \frac{P_1 - P_2 - 0}{\sqrt{\frac{P_1(1-P_1)}{n_1} + \frac{P_2(1-P_2)}{n_2}}} \approx -1.001$$

Since the critical value for a one-sided test with error level 5% is -1.64 (-1.65, -1.645 also accepted), we can't reject the null hypothesis. The proportion of model 2 cars that need major repairs within 2 years is not discernibly higher than that of model 1 cars.
Can also do this using 1-sided 95% confidence interval:

$$\begin{aligned}
\mu &< (P_1 - P_2) + (1.64)\sqrt{\frac{P_1(1-P_1)}{n_1} + \frac{P_2(1-P_2)}{n_2}} \\
\mu &< 0.015
\end{aligned}$$

Since 0 falls within the interval, we can't reject the null hypothesis.