Name: ______________________________

This exam has 5 problems. Each problem is worth 20 points. Points are assigned to parts of a problem as indicated.

This exam is closed book, so please put books and notes on the floor, with the exception of an optional single standard-size formula sheet. You may use a calculator, but you can't share one. A table of the standard normal distribution, a table of some common densities and probability mass functions, and two sheet of scratch paper are included at the end of the exam.

Please show your work, and indicate your answers in the boxes provided. If you need more space for calculations, use the back of the sheet on which the problem appears. If that fills up, then use the scratch pages.

Good Luck!

Please sign the Duke Honor Code:

I have neither given nor received unauthorized aid on this examination.

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signature

Problem	Points
1	(/20)
2	(/20)
3	(/20)
4	(/20)
5	(/20)

1. (20 points) Each of 100 polycarbonate plastic disks in a lot is tested for scratch resistance and shock resistance. The results of these tests are tabulated below:

		Shock Resistance	
		High	Low
Scratch	High	76	12
Resistance	Low	10	2

After testing, one of these disks is chosen at random. Using this information, answer the following questions:

(a) (5 points) What is the probability that the disk has high shock resistance?

(b) (5 points) What is the probability that the disk has both high shock and high scratch resistance?

(c) (5 points) You are given that the disk has high scratch resistance. What it the probability that it also has high shock resistance?

(d) (5 points) Are the events "Disk has high scratch resistance" and "Disk has high shock resistance" independent? Explain your answer.

2. (20 points) In the following situations, an outcome can usefully be modeled by a random variable. For each, indicate which type of random variable is **most appropriate**. (You can assume independence where this is convenient; for example, for the days on which it rains can be assumed to be independent.)

   (a) (5 points) The number of rainy days in the month of March in Durham.
   (a) Poisson (b) binomial (c) geometric (d) uniform (e) exponential (f) normal

   (b) (5 points) The number of years between 20 inch snowfalls in Durham.
   (a) Poisson (b) binomial (c) geometric (d) uniform (e) exponential (f) normal

   (c) (5 points) The concentration of cadmium in a sample of sediment from a polluted river.
   (a) Poisson (b) binomial (c) geometric (d) uniform (e) exponential (f) normal

   (d) (5 points) The number of typos in each 100 pages of an encyclopedia. (a) Poisson (b) binomial (c) geometric (d) uniform (e) exponential (f) normal

3. (20 points) A communications satellite is launched into orbit on an three-stage rocket. The first stage of the rocket fails with probability 0.10, the second and third stages are more reliable, each failing with probability 0.01. The satellite itself has a rocket, which places it into geosynchronous orbit, failing with probability 0.001. Assuming that all stages of the rocket, and the satellite itself, fail (and succeed) independently:

   (a) (4 points) What is the probability that the satellite is successfully placed into the desired orbit?

   (b) (4 points) Given that the first stage works successfully, what is this success probability?

   (c) (4 points) Given that the first stage does not fail, what is this probability that the second stage does not fail?

   (d) (8 points) You are told that the satellite did not make it into orbit, but nothing more. What is the probability that the first stage failed?

4. (20 points) The number of cracks in a section of interstate highway which are large enough to require repair is assumed to be a Poisson random variable with parameter $\lambda = 1$ crack per mile.

   (a) (5 points) What is the probability that no cracks require repair in 5 miles of highway?

   (b) (5 points) What is the probability that more than one crack requires repair in 5 miles of highway?

   (c) (5 points) The highway is inspected and repaired in one mile sections. What is the probability that no cracks are found until mile 5?

   (d) (5 points) What is the probability that 1 of the first 5 sections inspected has cracks, but the other 4 do not?

5. (20 points) The life of a semiconductor laser is normally distributed with a mean of 7000 hours and a standard deviation of 600 hours.

   (a) (5 points) What is the probability that a laser lasts less than 5000 hours?

   (b) (5 points) What is the probability that a laser lasts more than 9000 hours?

   (c) (5 points) What is the lifetime in hours exceeded by 99% of all lasers?

   (d) (5 points) Seven lasers are required in an industrial system. What is the probability that all seven function for at least 5000 hours?

Probability Distributions:

Bernoulli	$f_X(1) = p; f_X(0) = 1-p$ $\mu = p,\ \sigma^2 = p(1-p)$
Binomial	$f_X(x) = \left(\begin{array}{c} n \\ x \end{array}\right) p^x(1-p)^{n-x}, x = 0,\ldots,n$ $\mu = np,\ \sigma^2 = np(1-p)$
Poisson	$f_X(x) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0,1,\ldots,\ (0! = 1)$ $\mu = \lambda,\ \sigma^2 = \lambda$
Geometric	$f_X(x) = p(1-p)^{x-1}, x = 1,2,\ldots$ $\mu = 1/p,\ \sigma^2 = (1-p)/p^2$
Uniform	$f_X(x) = 1/(b-a), a \leq x \leq b$ $F_X(x) = (x-a)/(b-a)$ $\mu = (a+b)/2,\ \sigma^2 = (b-a)^2/12$
Exponential	$f_X(x) = \lambda e^{-\lambda x}, 0 \leq x < \infty$ $F_X(x) = 1 - e^{-\lambda x}$ $\mu = 1/\lambda,\ \sigma^2 = 1/\lambda^2$
Standard Normal	$f_X(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}}, -\infty < x < \infty$ $F_X(x) = \Phi(x)$, which is tabulated. $\mu = 0,\ \sigma^2 = 1$
Normal	$f_X(x) = \frac{e^{\frac{-(x-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}, -\infty < x < \infty$ $F_X(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$ $\mu = \mu,\ \sigma^2 = \sigma^2$

Conditional Probability and Bayes Theorem:

$$\begin{aligned} \mathrm{P}(A|B) &= \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)} \\ \mathrm{P}(B|A) &= \frac{\mathrm{P}(A|B)\mathrm{P}(B)}{\mathrm{P}(A|B)\mathrm{P}(B) + \mathrm{P}(A|B')\mathrm{P}(B')} \end{aligned}$$

Expectation and Variance:

$$\begin{aligned} \mu &= E(X) = \sum_x x f_X(x) \text{ or } \int_{-\infty}^{\infty} x f_X(x)dx \\ \sigma^2 &= V(x) = \sum_x (x-\mu)^2 f_X(x) \text{ or } \int_{-\infty}^{\infty} (x-\mu)^2 f_X(x)dx \\ &= E(X^2) - \mu^2 \end{aligned}$$

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