Name: ______________________________

This exam has 5 problems. Each problem is worth 20 points. Points are assigned to parts of a problem as indicated.

This exam is closed book, so please put books and notes on the floor, with the exception of an optional single standard-size formula sheet. Also, two pages of some basic formulas is provided at the end of the exam. You may use a calculator, but you can't share one. Tables of the standard normal distribution, Student-t distribution, and two sheet of scratch paper are included at the end of the exam.

Please show your work. If you write an answer in a space other than the space provided for that answer, please label it! If you need more space for calculations, use the back of the sheet on which the problem appears, or else use the scratch pages.

Good Luck!

Please sign the Duke Honor Code:

I have neither given nor received unauthorized aid on this examination.

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signature

Problem	Points
1	(/20)
2	(/20)
3	(/20)
4	(/20)
5	(/20)

1. (a) (8 points) In a particular hypothesis test, we reject the null hypothesis for large values of a test statistic T. We observe a test statistic $T = 4$ and a P-value of 0.07. In words, what does this P-value mean?

(b) (6 points) You're a contestant on the TV show *Millionaire*. It's time for the million-dollar question. You've used up your "life-line" and "50-50", and Regis is waiting for your final answer:

A 95% confidence interval for a normal mean μ *is* $[-2, 1]$. *Given this information, consider the following four statements about a test of the hypothesis* $H_0 : \mu = 0$ *against* $H_1 : \mu \neq 0$ *at the* $\alpha = 0.05$ *level. Exactly one of these statements is true; circle the appropriate letter:*

(a) H_0 is rejected, the P-value is less than 0.05.

(b) H_0 is rejected, the P-value is greater than 0.05.

(c) H_0 is not rejected, the P-value is less than 0.05.

(d) H_0 is not rejected, the P-value is greater than 0.05.

(c) (6 points) Assume that thicknesses of playing cards are normally distributed with a mean of 0.25mm and a standard deviation of 0.005mm. What is the distribution, mean, and standard deviation of the thicknesses of decks of 52 cards?

2. The joint probability mass function of two random variables, X and Y is specified by the following table:

x	y	$f_{X,Y}(x,y)$
1	1	0.2
1	2	0.2
1	3	0.2
2	1	0.2
2	2	0.1
3	2	0.1

(a) (8 points) What is $E(X)$?

(b) (8 points) What is $E(Y)$?

(c) (4 points) Are X and Y independent? (Explain your answer.)

3. A state is considering raising the maximum speed limit from 55mph to 65mph. A survey is done to see what people think about this. Of $n_1 = 300$ urban residents polled, $x_1 = 63$ are in favor of the proposal. Among rural residents, the corresponding values are $n_2 = 180$ and $x_2 = 50$. Assume that the number of respondents favoring the proposal has a binomial distribution, with parameters (p_1, n_1) and (p_2, n_2) for urban and rural, respectively.

   (a) (12 points) Test the hypothesis $H_0 : p_1 = p_2$ vs the alternative $H_1 : p_1 \neq p_2$. Let $\alpha = 0.05$. Show the test statistic, region for rejecting H_0, and conclusion of the test.

   (b) (8 points) What is the P-value for this test?

4. The heart rates of $n = 9$ individuals were measured both after smoking a low dose of marijuana and after smoking tobacco. Assume that the heart rate difference for each individudal is normally distributed with mean μ. The sample mean and standard deviation of these differences are $\bar{x} = 8.2$ and $s = 12.8$.

   (a) (12 points) Test $H_0 : \mu = 0$ against $H_1 : \mu > 0$. Use $\alpha = 0.05$. Show the test statistic, critical region, and test conclusion.

   (b) (8 points) Construct a 95% lower confidence interval for μ.

5. Soil Ph was measured for $n = 9$ sample at each of two locations. The sample means and standard deviations are $\bar{x}_1 = 8.03$, $\bar{x}_2 = 7.44$, $s_1 = 0.29$, $s_2 = 0.22$. Assume that Ph is normally distributed, with means μ_1 and μ_2 for the two locations.

   (a) (12 points) Test the hypothesis $H_0 : \mu_1 = \mu_2$ against the alternative $H_1 : \mu_1 \neq \mu_2$, with $\alpha = 0.05$. Show the test statistic, region for rejection of H_0, and conclusion of your test. (To keep things simple, you may assume that the unknown variances are equal.)

   (b) (8 points) Construct a 95% two-sided confidence interval for $\mu_1 - \mu_2$.

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