Name: _____

This exam has 5 problems. Each problem is worth 20 points. Points are assigned to parts of a problem as indicated.

This exam is closed book, so please put books and notes on the floor, with the exception of an optional single standard-size formula sheet. Also, two pages of some basic formulas is provided at the end of the exam. You may use a calculator, but you can't share one. Tables of the standard normal distribution, Student-t distribution, and two sheet of scratch paper are included at the end of the exam.

Please show your work. If you write an answer in a space other than the space provided for that answer, please label it! If you need more space for calculations, use the back of the sheet on which the problem appears, or else use the scratch pages.

Good Luck!

Please sign the Duke Honor Code:

I have neither given nor received unauthorized aid on this examination.

_____

signature

Problem	Points
1	(/20)
2	(/20)
3	(/20)
4	(/20)
5	(/20)

1. A chemist is interested in the concentration of glucose in human serum. She makes 10 independent glucose measurements, on samples of serum which can each be assumed to have the same mean concentration μ. The average of these measurements (in micrograms per ml) is $\bar{x} = 78$. From extensive previous work, the chemist knows that the standard deviation of the distribution of a measurement is $\sigma = 3$.

   (a) (6 points) What is the approximate sampling distribution of the random variable $\bar{X}$, which corresponds to the average of the measurements? Provide the distribution type (normal, binomial, etc.), mean, and standard deviation.

   (b) (4 points) We do not know the precise population from which these measurements are a random sample. Why do you think the approximation that you used above is likely to be a good one?

   (c) (8 points) What is the P-value of a test of the null hypothesis $H_0 : \mu = 75$ against the alternative $H_1 : \mu > 75$.

   (d) (2 points) For the above hypothesis test, what is the smallest Type I error (α) for which H_0 can be rejected?

2. A total of 30 measurements are made on cyanide in soil. The sample mean and standard deviation of these measurements are $\bar{x} = 95$mg/kg and $s = 20$mg/kg.

   (a) (10 points) Construct a 90% two-sided confidence interval for μ, the population mean concentration of cyanide.

   (b) Perform the hypothesis test for which $H_0 : \mu = 100$, and $H_1 : \mu < 100$. Use $\alpha = 0.05$.

      i. (3 points) What is the observed standardized test statistic under H_0?

      ii. (5 points) For what values of this statistic should you reject the null hypothesis?

      iii. (2 points) Can you reject the null hypothesis for this problem?

3. The breaking strength of a yarn used in manufacturing drapery material is required to be at least 100psi. A random sample of 9 specimens in tested, and the average breaking strength is found to be $\bar{x} = 98$psi. From past experience, we know that the standard deviation of breaking strength for this yarn is $\sigma = 2$psi. We are interested in hypothesis tests and confidence intervals for the population mean breaking strength, μ.

   (a) Test the hypothesis $H_1 : \mu > 100$, where the null hypothesis is $H_0 : \mu = 100$. Use $\alpha = 0.05$.

      i. (4 points) What is the observed standardized test statistic under the null hypothesis?

      ii. (4 points) What is the critical region for rejecting the null hypothesis?

      iii. (2 points) Can you reject the null hypothesis for this problem?

   (b) (10 points) Find a 95% two-sided confidence interval for μ.

4. Scientists believe that frogs eggs are less likely to hatch in sunlight than in shade, because of damage done by ultraviolet radiation. California tree frogs, on the other hand, have an enzyme which is believed to protect the eggs from the damaging effects of sunlight. An experiment was conducted with California tree frogs. Of 70 eggs in the shade, 34 hatched. Of 80 eggs in the sun, 31 hatched.

   Let p_1 be the probability that an egg hatches in sunlight, and let p_2 be the probability that it hatches in shade. Assume that the number of eggs hatching are binomial random variables, and use the normal approximation to the binomial distribution.

   (a) i. (5 points) Estimate the difference in probabilities $p_2 - p_1$ ("shade − sun")

   ii. (5 points) What is the standard error of this estimate?

   (b) (8 points) Construct a 95% (one-sided) lower confidence interval for $p_2 - p_1$.

   (c) (2 points) Based on this confidence interval, do you think that one should conclude that eggs are more likely to hatch in shade than in sun?

5. Wear tests were performed on rubber for an automotive application which is manufactured by two different companies. The decrease in mass due to wear of 25 samples from each manufacturer was measured.

   For manufacturer 1, the sample mean and standard deviation of mass loss due to wear are $\bar{x}_1 = 20$ and $s_1 = 2$, respectively. For manufacturer 2, the corresponding results are $\bar{x}_2 = 15$ and $s_2 = 8$. When answering the questions below, assume that the population variances for the mass losses for the two manufacturers are unknown and unequal.

   (a) i. (2 points) Estimate the difference $\mu_1 - \mu_2$ of the mass loss for the two manufacturers.

   ii. (3 points) What is the standard error of this estimate?

   (b) Test the hypothesis $H_0 : \mu_1 = \mu_2$ against the alternative $H_1 : \mu_1 \neq \mu_2$.

   i. (3 points) What is the observed standardized test statistic under H_0?

   ii. (10 points) For what values of this statistic should you reject H_0?

   iii. (2 points) Can you reject H_0 for this problem?

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