Name: ______________________________

This exam has 4 problems. Each problem is worth 25 points. Points are assigned to parts of a problem as indicated.

This exam is closed book, so please put books and notes on the floor, with the exception of an optional single standard-size formula sheet. Also, a page of some basic formulas is provided at the end of the exam. You may use a calculator, but you can't share one. A table of the Student-t distribution and two sheet of scratch paper are included at the end of the exam.

Please show your work. If you write an answer in a space other than the space provided for that answer, please label it! If you need more space for calculations, use the back of the sheet on which the problem appears, or else use the scratch pages.

Good Luck!

Please sign the Duke Honor Code:

I have neither given nor received unauthorized aid on this examination.

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signature

Problem	Points
1	/25
2	/25
3	/25
4	/25

1. Answer the following multiple choice questions by circling the letter corresponding to the most appropriate response. Each is worth 6 points; you get one additional point for free.

   (a) (6 points) In order to test the effectivness of a physical education program, the time that it took each of 25 fourth grade students to run 100 meters was measured before and after they participated in the program. Which of the following is most appropriate for analyzing these data.

   - (a) simple linear regression,
   - (b) a two-sample t-test with pooled variance,
   - (c) a two-sample t-test allowing for unequal variances,
   - (d) a paired t-test.

   (b) (6 points) When testing for the statistical significance of an individual coefficient in a regression model which of the following is most useful

   - (a) the residual standard deviation $(\hat{\sigma})$,
   - (b) plots of residuals against each independent variable in the model,
   - (c) the coefficient of determination (R^2),
   - (d) the t-ratio of the estimated coefficient to its estimated standard error.

   (c) (6 points) To check the adequacy of a multiple regression model, which of the following is/are useful

   - (a) the residual standard deviation $(\hat{\sigma})$,
   - (b) plots of residuals against each independent variable in the model,
   - (c) the F-test of the hypothesis that all coefficients are equal to zero except the intercept β_0,
   - (d) all of the above.

   (d) (6 points) In multiple regression, the ratio of the regression mean square to the mean square due to error is

   - (a) the coefficient of determination (R^2),
   - (b) a statistic which has an F-distribution when the null hypothesis that all of the coefficients except the intercept (β_0) equals zero is true,
   - (c) an estimate of the residual standard deviation $\hat{\sigma}$.
   - (d) an F-statistic for testing the hypothesis that all of the coefficients, *including* the constant intercept (β_0) are equal to zero.

2. The knowledge of basic statistical concepts of 10 engineers selected at random was measured on a scale of 0 to 100, before and after they took a statistics short course. Each engineer was tested before and after, and we would like to test the hypothesis $H_0 : \mu_1 = \mu_2$ that the population mean is the same before and after the course, against the alternative $H_1 : \mu_2 > \mu_1$, that the engineers improved after the course. The data are as follows

Engineer	Before (1)	After (2)	Difference (D)
1	43	51	8
2	82	84	2
3	77	74	-3
4	39	48	9
5	51	53	2
6	66	61	-5
7	55	59	4
8	61	75	14
9	79	82	3
10	43	48	5

The means and standard deviations for each test and for the differences are: $\bar{x}_1 = 59.6$, $s_1 = 15.97$, $\bar{x}_2 = 63.5$, $s_2 = 14.06$, $\bar{x}_D = 3.9$, $s_D = 5.57$.

(a) (7 points) Since the population variances are not known for this problem, one might consider using one of three two-sample t-tests: unpaired, pooled variance; unpaired, unequal variances; and paired. Which do you think is most appropriate, and why?

(b) (10 points) What is the observed test statistic for your chosen test?

(c) (5 points) Can you reject H_0 at the $\alpha = 0.05$ level?

(d) (3 points) Based on this analysis, would you conclude that the engineer's understanding of statistics has improved?

3. A rubber manufacturer is interested in understanding how hardness and tensile strength relate to abrasion of rubber. Data were collected on each of these properties for $n = 32$ specimens, and the following multiple regression model was fit:

$$Y_i = \text{ABRASION}_i = \beta_0 + \beta_1 \text{HARDNESS}_i + \beta_2 \text{STRENGTH}_i + \epsilon_i$$

Least squares estimates of the coefficients and their estimated standard errors are as follows:

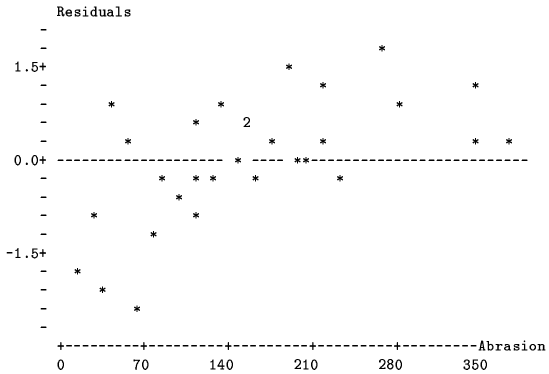
```
Predictor        Coef        Stdev
Constant       885.16        61.75
hardness      -6.5708       0.5832
strength      -1.3743       0.1943
s = 36.49        R-sq = 84.0%
```

(a) (8 points) Is hardness a statistically significant predictor ($\alpha = 0.05$) of abrasion? Perform an appropriate statistical test.

(b) (8 points) Is tensile strength a statistically significant predictor ($\alpha = 0.05$) of abrasion? Perform an appropriate statistical test.

(c) (9 points) Assuming that the model is correct, construct a 95% two-sided confidence interval for the change in abrasion which is caused by a unit change in hardness.

4. (a) (7 points) The residuals of the fit for the model from Problem 3 are displayed below. Based on this plot, do you think that the regression model is adequate, or do you think that it might be improved by adding additional regressors?



(b) (18 points) Below are the information on the fitted value for one of the observations, along with the estimated standard error of the fit. From the computer output from the previous problem, we have $\hat{\sigma} = 36.49$.

Obs.	abrasion	Fit	Stdev.Fit	Residual
2	206.00	203.55	13.17	2.45

i. (9 points) Construct a 95% two-sided confidence interval for the mean response

$$\beta_0 + \beta_1 \text{HARDNESS}_2 + \beta_2 \text{STRENGTH}_2.$$

ii. (9 points) Construct a 95% two-sided prediction interval for a future measure of abrasion on a rubber sample having the same values of hardness and tensile strength as the observation described in the above output.

Official STAT-113 Fomula Sheet (Collect them All!)

- Regression analysis:
  - Model and Fitted Values:

$$\begin{aligned} y_i &= \beta_0 + \beta_1 x_{i1} + \ldots + \beta_k x_{ik} + \epsilon_i \\ \hat{y}_i &= \hat{\beta}_0 + \hat{\beta}_1 x_{i1} + \ldots + \hat{\beta}_k x_{ik} \end{aligned}$$

  - Residuals: $e_i = y_i - \hat{y}_i$.
  - Estimated standard errors of etimates: $\mathrm{SE}\,(\hat{\beta}_j)$.
  - Residual Standard Deviation:

$$s = \hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n} e_i^2}{n-k-1}}$$

  - Tests and Confidence Intervals for Coefficients:

$$t_0 = \frac{\hat{\beta}_j}{\mathrm{SE}\,(\hat{\beta}_j)}$$

    has a t-distribution with $n-k-1$ degrees of freedom.
  - Confidence intervals for mean response corresponding to ith observation

$$\hat{y}_i \pm t_{\alpha/2,n-k-1} \mathrm{SE}\,(\hat{y}_i)$$

  - Prediction interval

$$\hat{y}_i \pm t_{\alpha/2,n-k-1} \sqrt{\mathrm{SE}\,(\hat{y}_i)^2 + \hat{\sigma}^2}$$

- Paired-sample t-test:

$$t_0 = \frac{\bar{D}}{s_D/\sqrt{n}}$$

  has a t-distribution with $n-1$ degrees of freedom under H_0.
- Pooled sample t-test

$$t_0 = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_1-2}}}$$

  has a t-distribution with $n_1 + n_2 - 2$ degrees of freedom under H_0.

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