Bayes Factors

- Evidence in favor of Model 1 over Model 2 can be obtained using Bayes Factors
- Bayes Factors can be interpreted like odds
- 2 log(B12) is twice the log Bayes Factor for comparing model 1 to model 2
- 2 log(B12) is approximately BIC(M2) - BIC(M1)
- Positive values imply support for Model 1
- Can be used to compare un-nested models !

Strength of Evidence

2 log(B12)	**B12**	**Evidence against Model 2**
0 to 2	1 to 3	Not worth a bare mention
2 to 6	3 to 20	Positive
6 to 10	20 to 150	Strong
> 10	> 150	Very Strong

Example from last time...

Compare model B to AC :

2*log Bayes Factor = 159.72 - 158.93 = 0.79

the Bayes Factor or odds in favor of model B over AC = exp(0.79/2) = 1.5

model	p	df	resSS	MSE	adjR2	BIC	postprob
NULL	1	27	8100	300	0	163.04	0.0405
A	2	26	6240	240	0.2	160.12	0.1740
B	2	26	5980	230	0.23	158.93	0.3175
C	2	26	6760	260	0.13	162.36	0.0567
AB	3	25	5500	220	0.27	161.02	0.1112
AC	3	25	5250	210	0.3	159.72	0.2132
BC	3	25	5750	230	0.23	162.26	0.0597
ABC	4	24	5160	215	0.28	163.71	0.0290

Problems with Model Selection

While model selection using BIC, adjusted R2, stepwise, forward, or backward selection leads to a single model, there are the following problems:

- several models may have similar values of adjusted R2 or BIC If they are equally good why pick one over the other?
- Each method may lead to a different "best model" Should we report all of them?
- Ignores Model Uncertainty...

Bayesian Model Averaging

- Rather than selecting a single model BMA uses all models, but models are weighted based on the support that they receive from the data as measured by the posterior model probabilities
- Posterior Model Probabilities

$$pr(M_j|Data) = \frac{pr(M_j)\exp(-.5\,BIC(M_j))}{\sum pr(M_k)\exp(-.5\,BIC(M_k))}$$

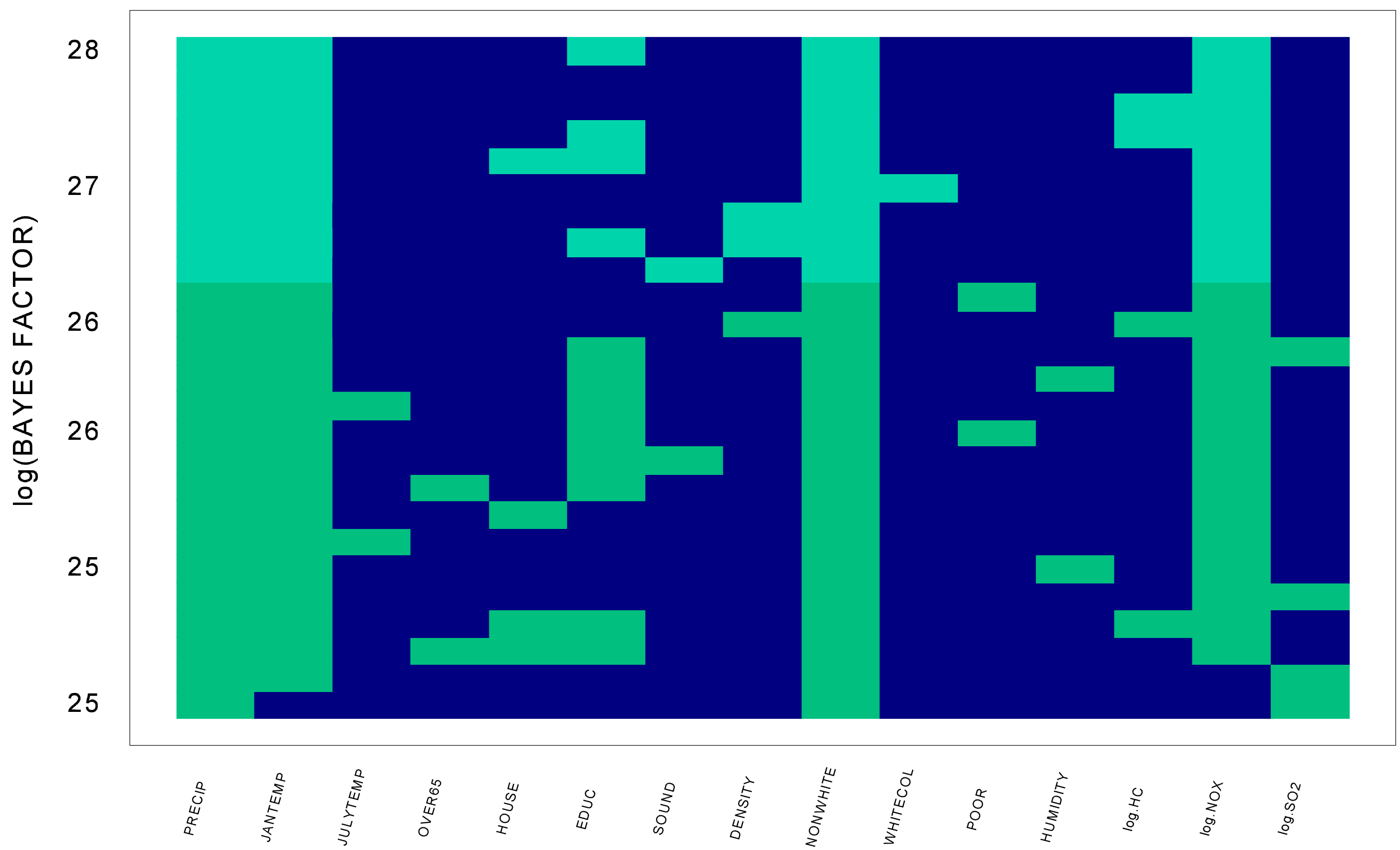
- Predictions:

$$(\hat{\mu}|Data) = \sum pr(M_j|Data)(\hat{\mu}|M_j, Data)$$

Pollution and Mortality Example

- Data from Ex 12.17
- measures of HC, NOX, SO2
- Question is pollution associated with mortality after adjusting for socio-economic and meteorological factors ?
- with 15 variables there are 2^15 models or
- 32,768 models
- Use BMA on a subset of models

Top 25 Models



Posterior Probabilities

- Posterior Probability that there is no pollution effect
- sum posterior probabilities of all models that do not include any of the pollution variables
- Posterior probability of NO EFFECT = 0.0043
- Odds in favor of an effect = 237
- For the energetic student, how does this compare to p-values?